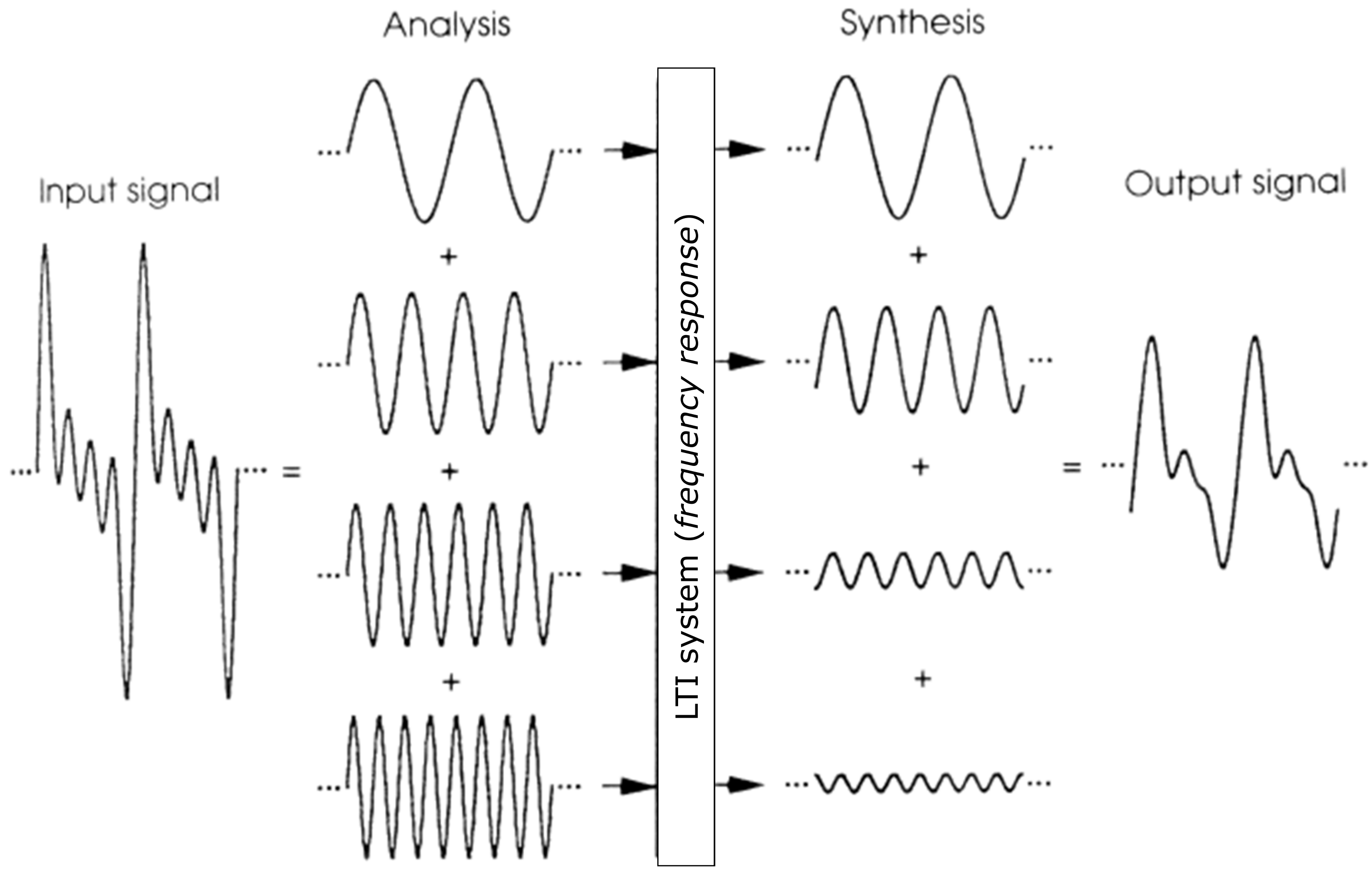
Signals, systems, acoustics and the ear

Week 4

Signals through Systems

Crucial ideas

- *Any* signal can be constructed as a sum of sine waves
- In a *linear time-invariant* (LTI) system, the response to a sinusoid is the same whether it is on its own, or as one component of a complex signal
  - No interaction of components
- An LTI system *never* introduces frequency components not present in the input
  - a sinusoidal input gives a sinusoidal output of the same frequency
- Hence, the output is the *sum* of the individual sinusoidal responses to each individual sinusoidal component of the input



waveform → spectrum *spectrum → waveform*

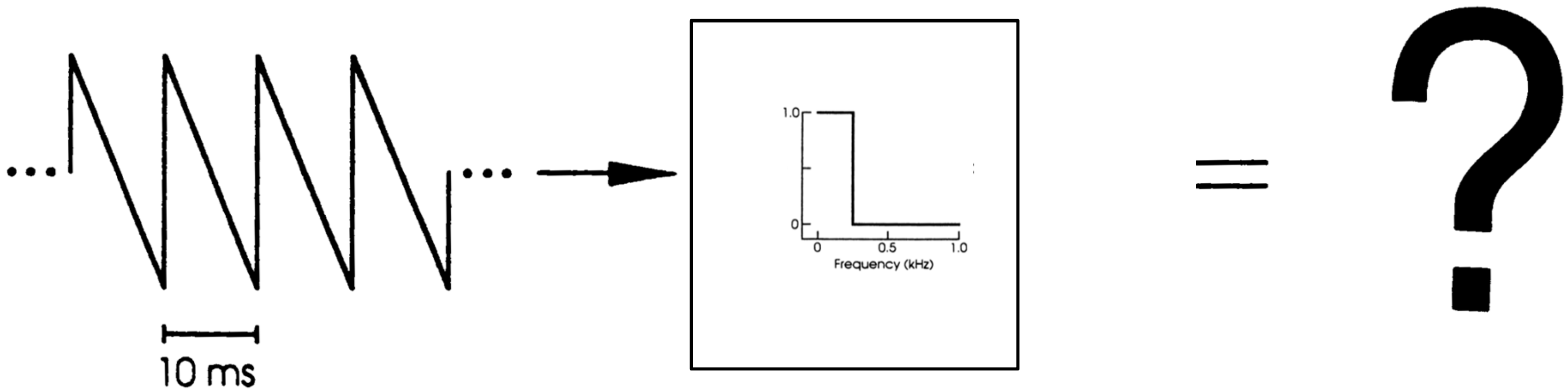
Fourier analysis Fourier synthesis

Six steps to determining system output to any particular input

1. Obtain the system's amplitude response
2. Obtain the system's phase response
3. Analyse the waveform to obtain its spectrum (amplitude and phase)
4. Calculate the output amplitude of each component sinusoid in the input spectrum
5. Calculate the output phase of each sinusoid
6. Sum the output component sinusoids

A particular example

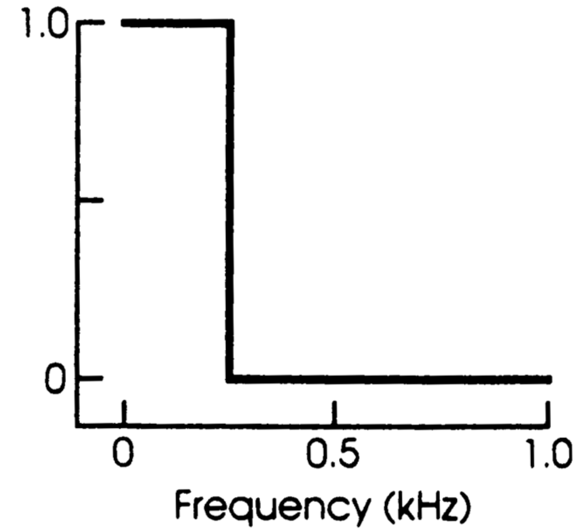
input signal — system — output signal



Step 1: Measure the system's response

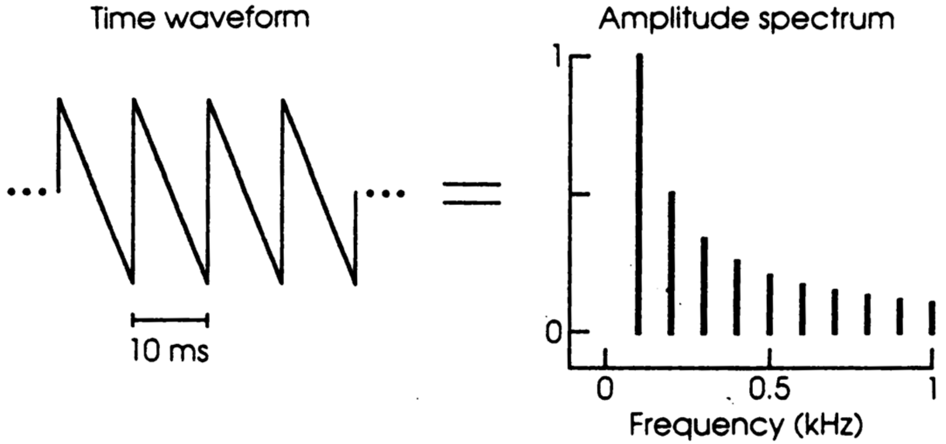
For example, by using sinewaves of different frequencies (as for the acoustic resonator)

Here the response has a gain of
- 1 for frequencies up to 250 Hz
- 0 for frequencies above 250 Hz



Assume phase response is a phase shift of zero degrees everywhere

Step 2: Sawtooth amplitude spectrum



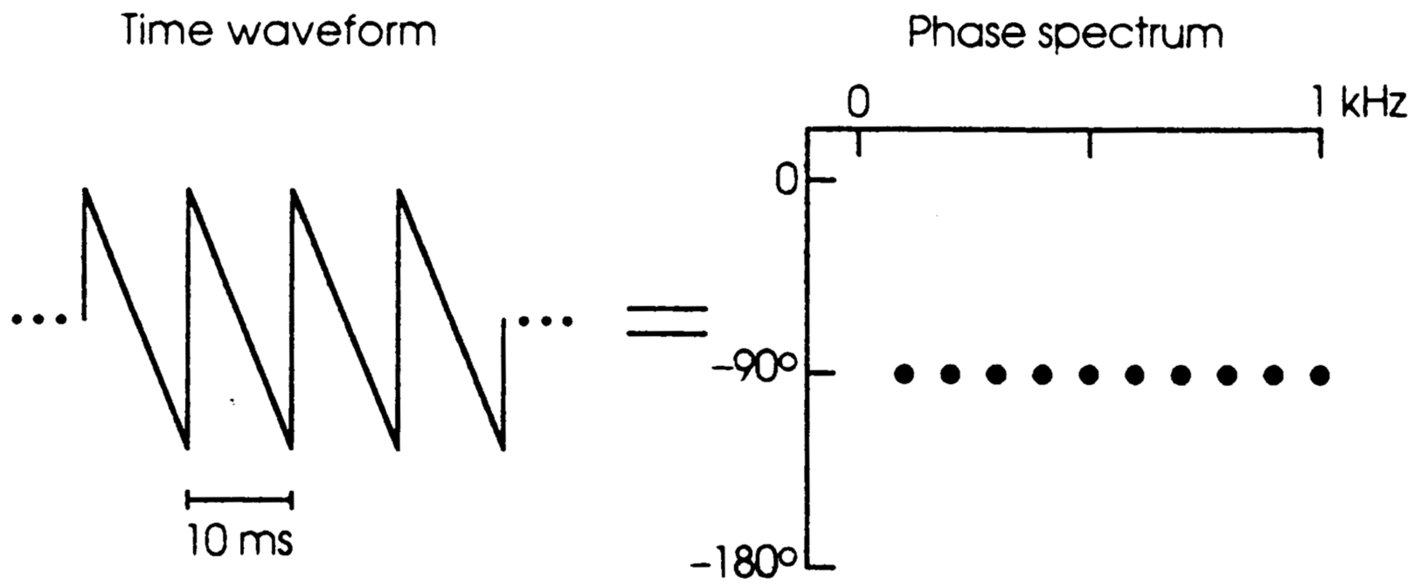
A(n) = A(1)/n

(*A* is the amplitude of a harmonic, index *n* is harmonic number)

A(1) is for this example 1 volt

Sawtooth phase spectrum

All components have a phase of -90^{o} (relative to a cosine)

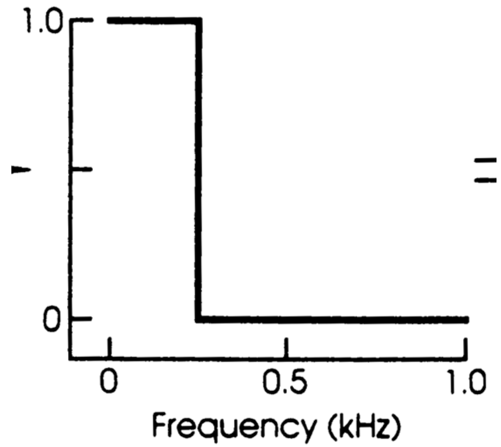


Remember!

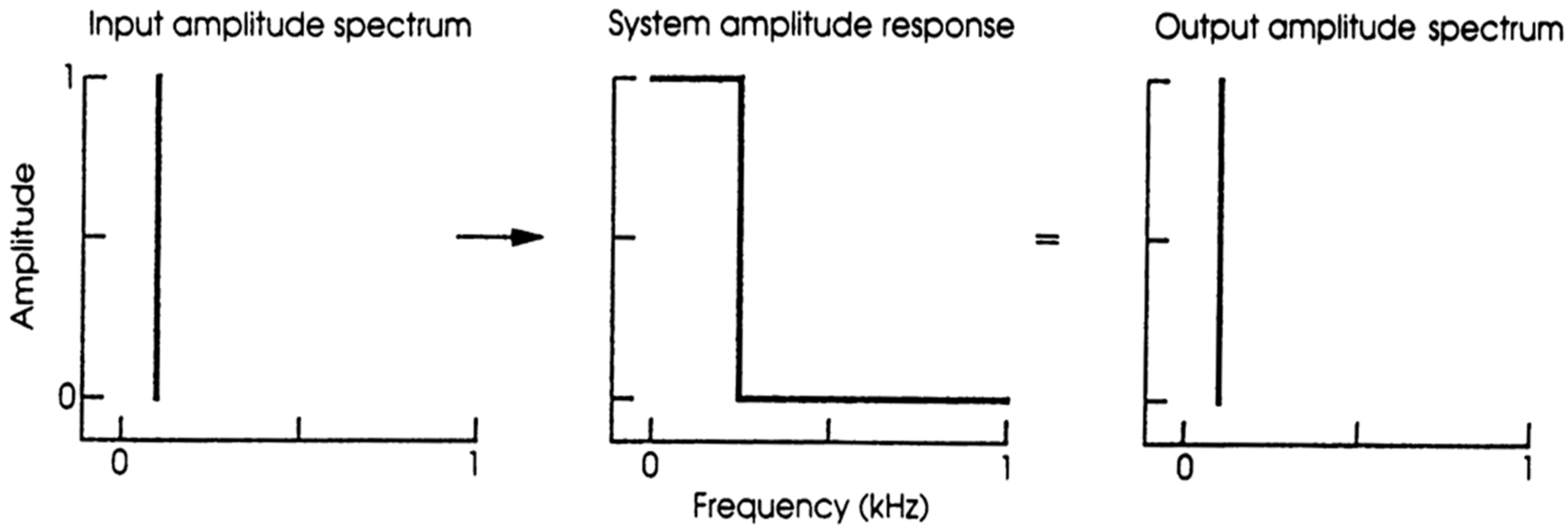
- Response = Output amplitude/Input amplitude
- So on linear scales …
  - Output amplitude = Response x Input amplitude
- But on dB (logarithmic) scales
  - Output amplitude = Response + Input amplitude
  - because log(a x b) = log(a) + log(b)
- For phase
  - Output phase = Response phase + Input phase

Response to harmonic 1 (100 Hz)

Input amplitude	Response gain	Output amplitude
1 V	1	?
Input phase	**Phase shift of response**	**Output phase**
-90	0	?



Graph of signal - system - output for harmonic 1

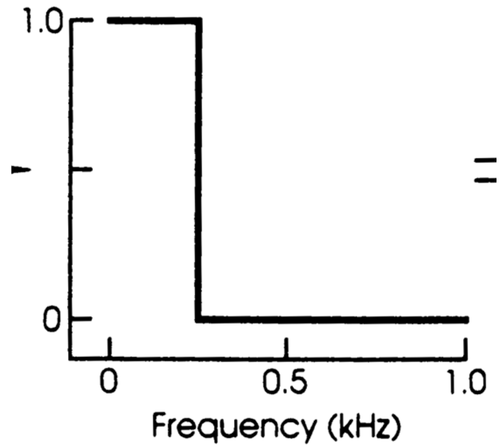


1 V x 1 = 1 V

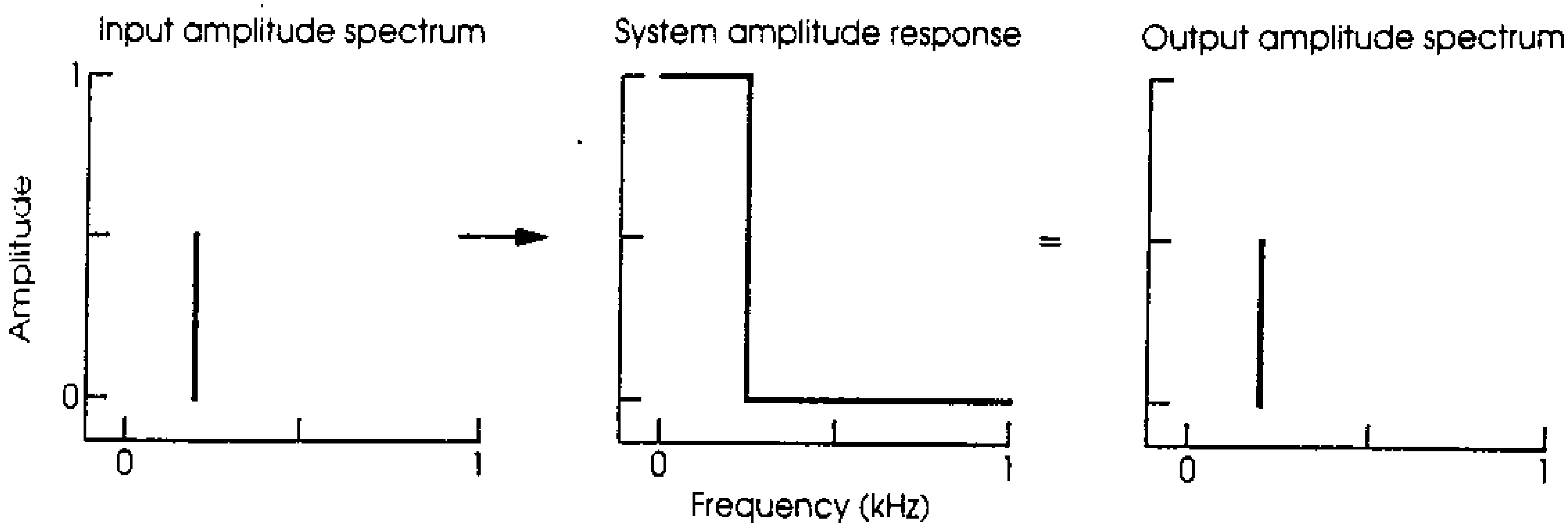
Response to harmonic 2 (200 Hz)

Input amplitude	Response gain	Output amplitude
1/2 V	1	?

Input phase	Phase shift of response	Output phase
-90	0	?



Graph of signal - system - output for harmonic 2

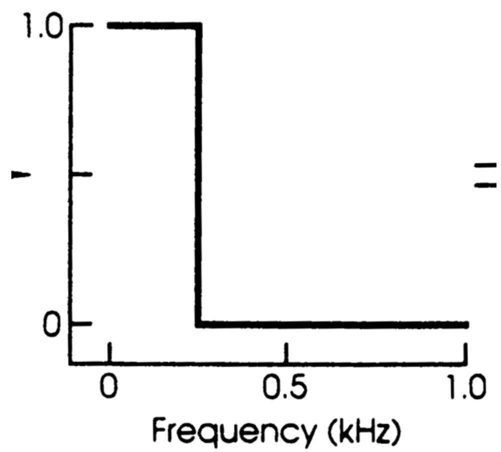


0.5 V x 1 = 0.5 V

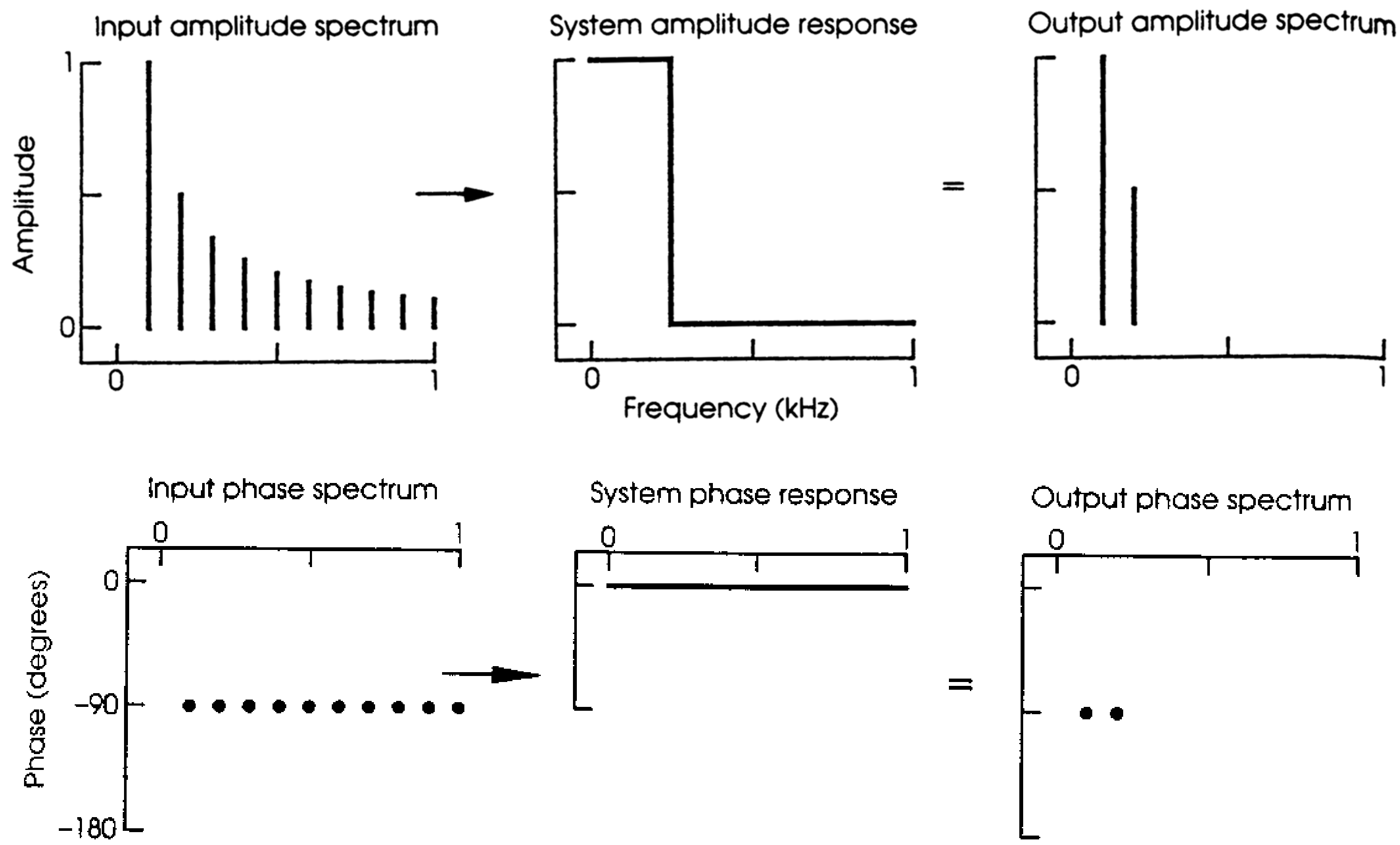
Response to harmonic 3 (300 Hz)

Input amplitude	Response gain	Output amplitude
1/3 V	0	?

Input phase	Phase shift of response	Output phase
-90	0	?



Response to whole signal



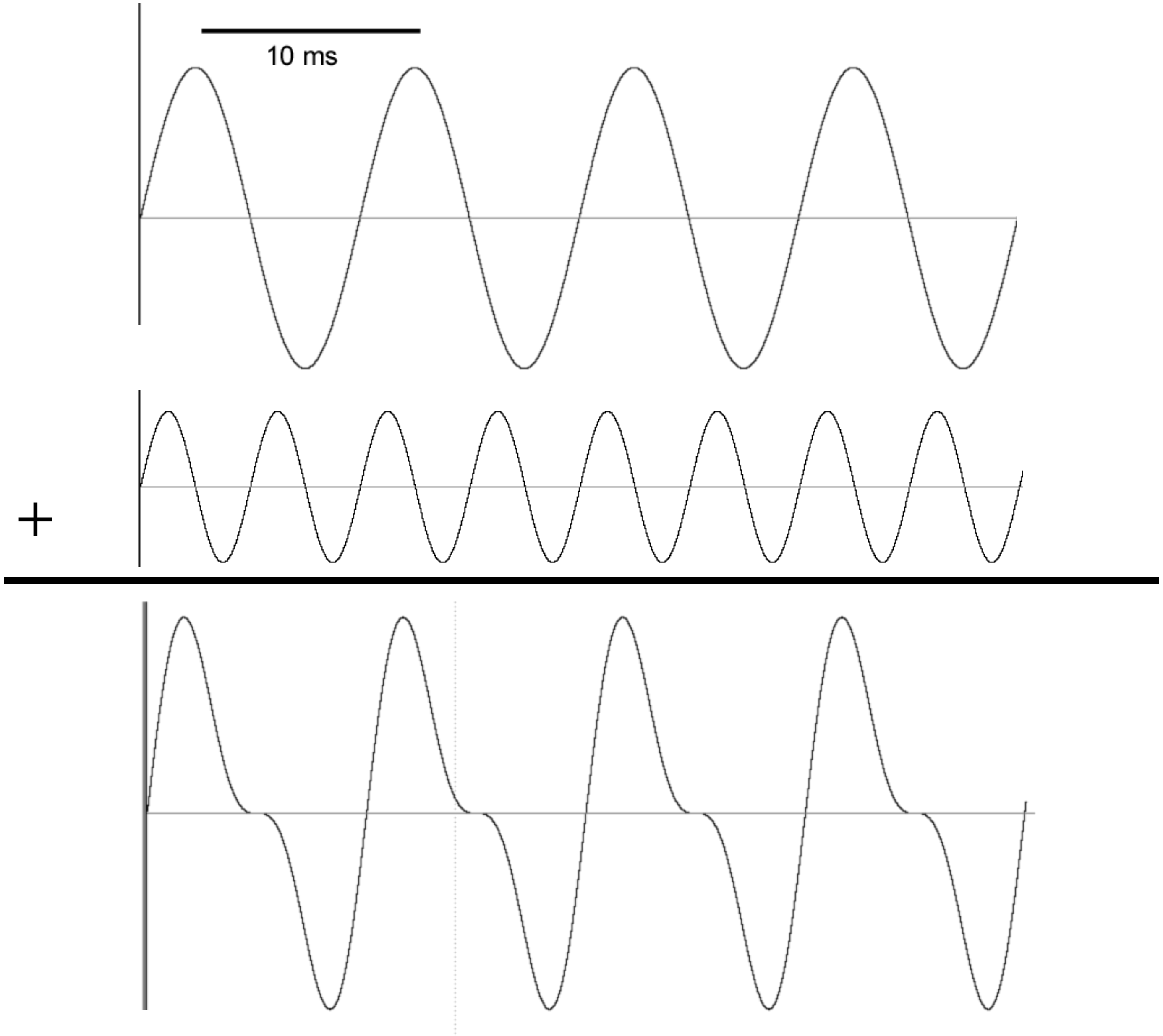
Waveform of output

100 Hz

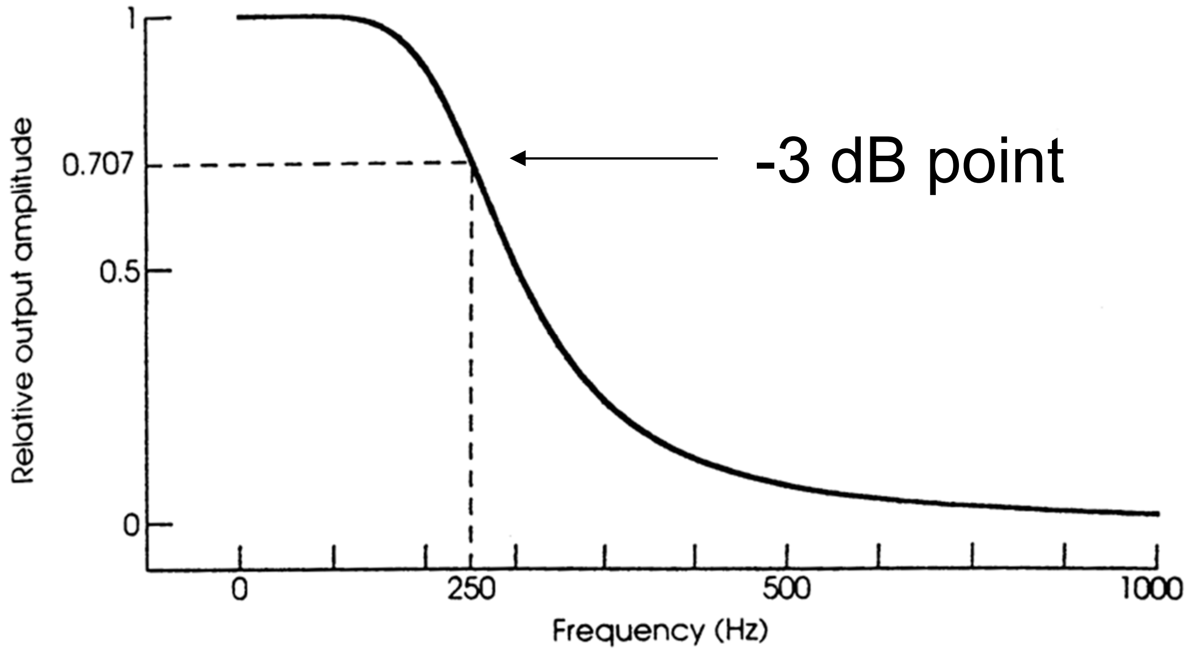
200 Hz

+

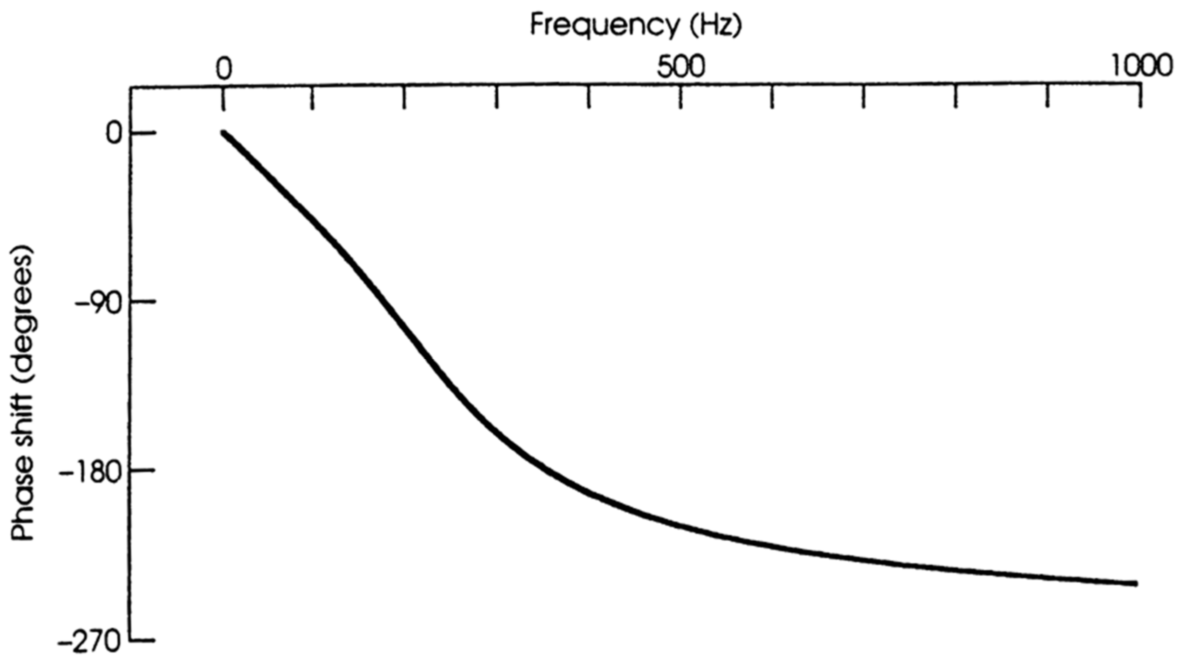
sum



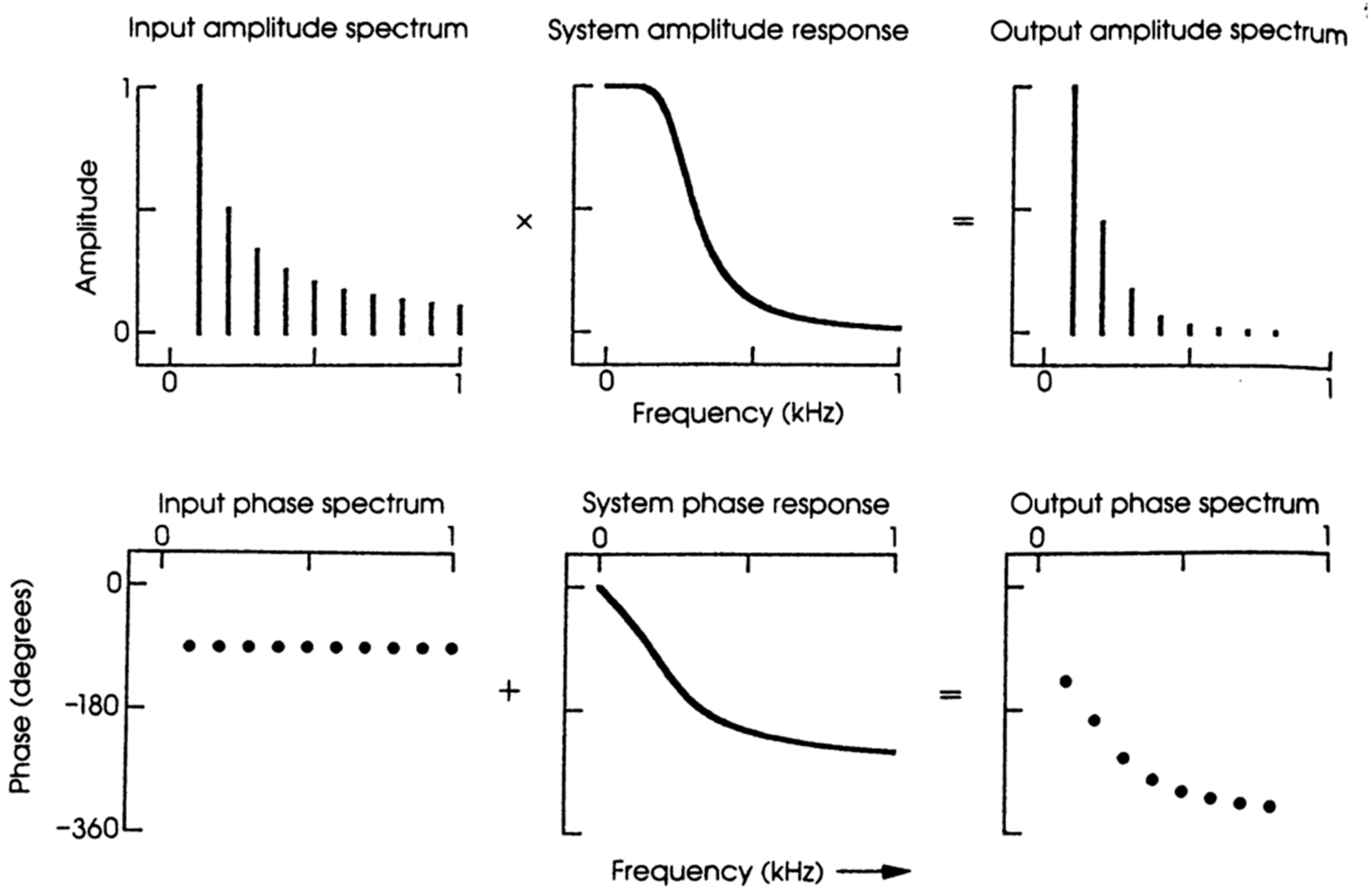
A realistic amplitude response



A phase response

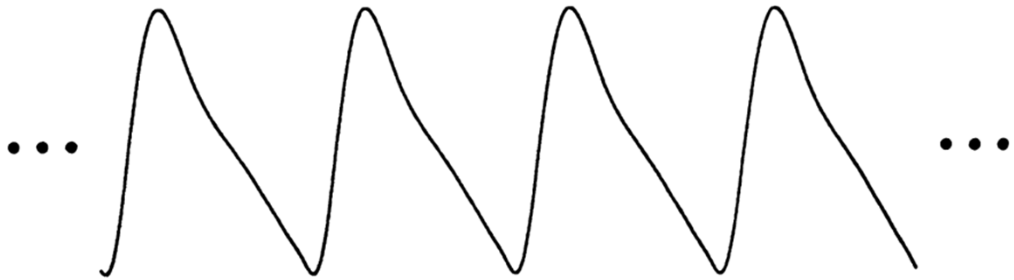


Sawtooth Wave:
Input - System - Output



Note that multiplications are done 'all at once'

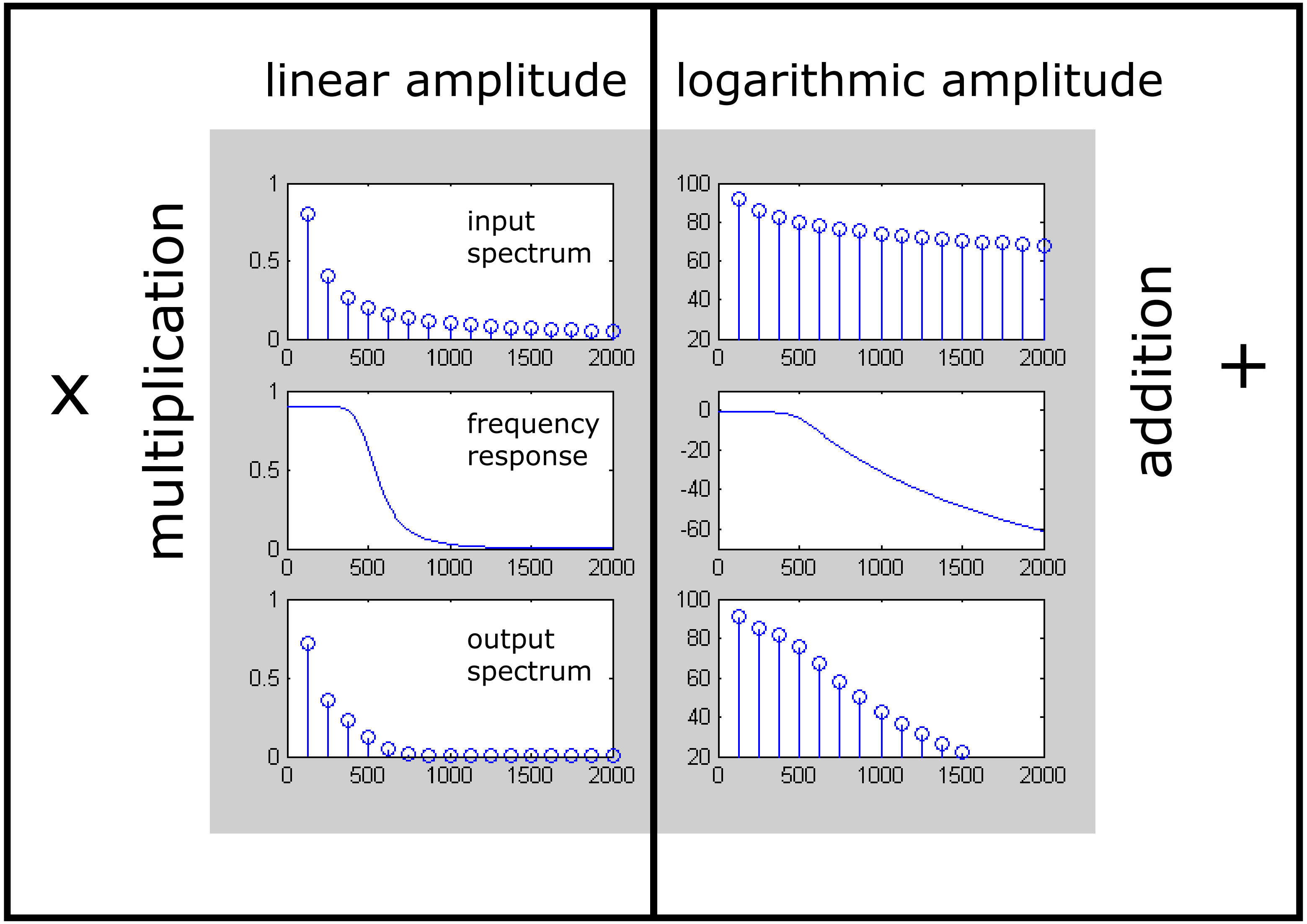
Output waveform



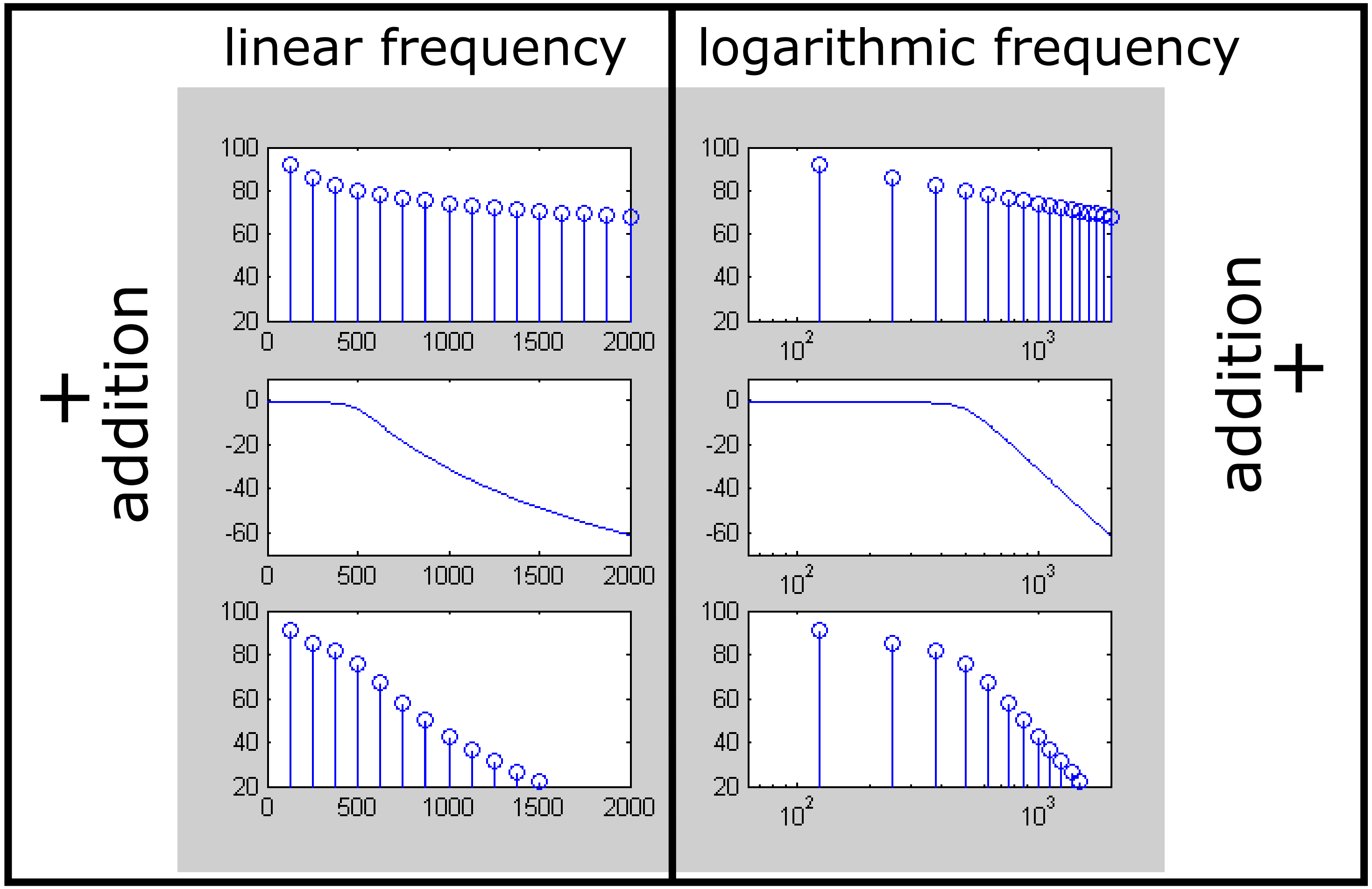
realistic lowpass filter

for comparison: ideal lowpass filter

Linear vs. logarithmic *amplitude* scales

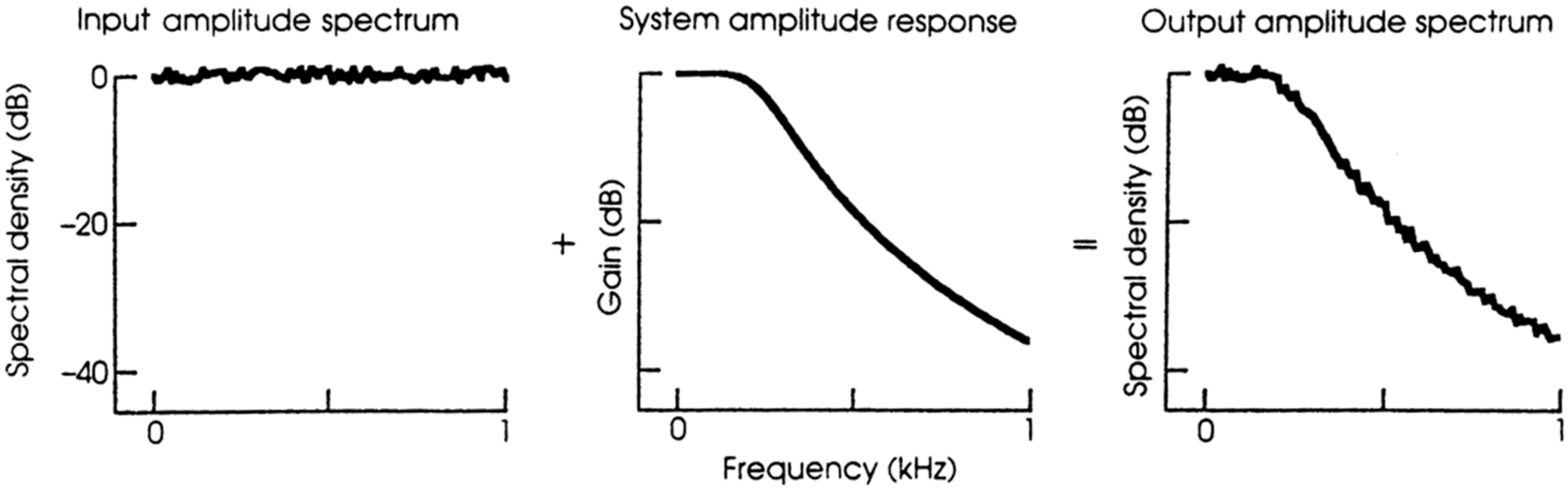


Linear vs. logarithmic *frequency* scales



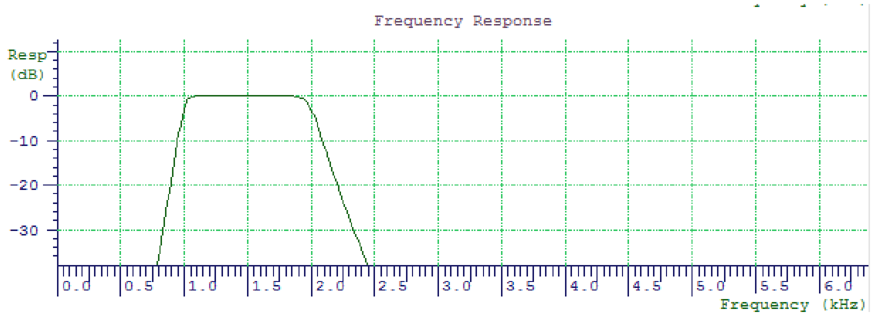
Logarithmic amplitude scales matter for calculations.
Logarithmic frequency scales are a matter of convenience.

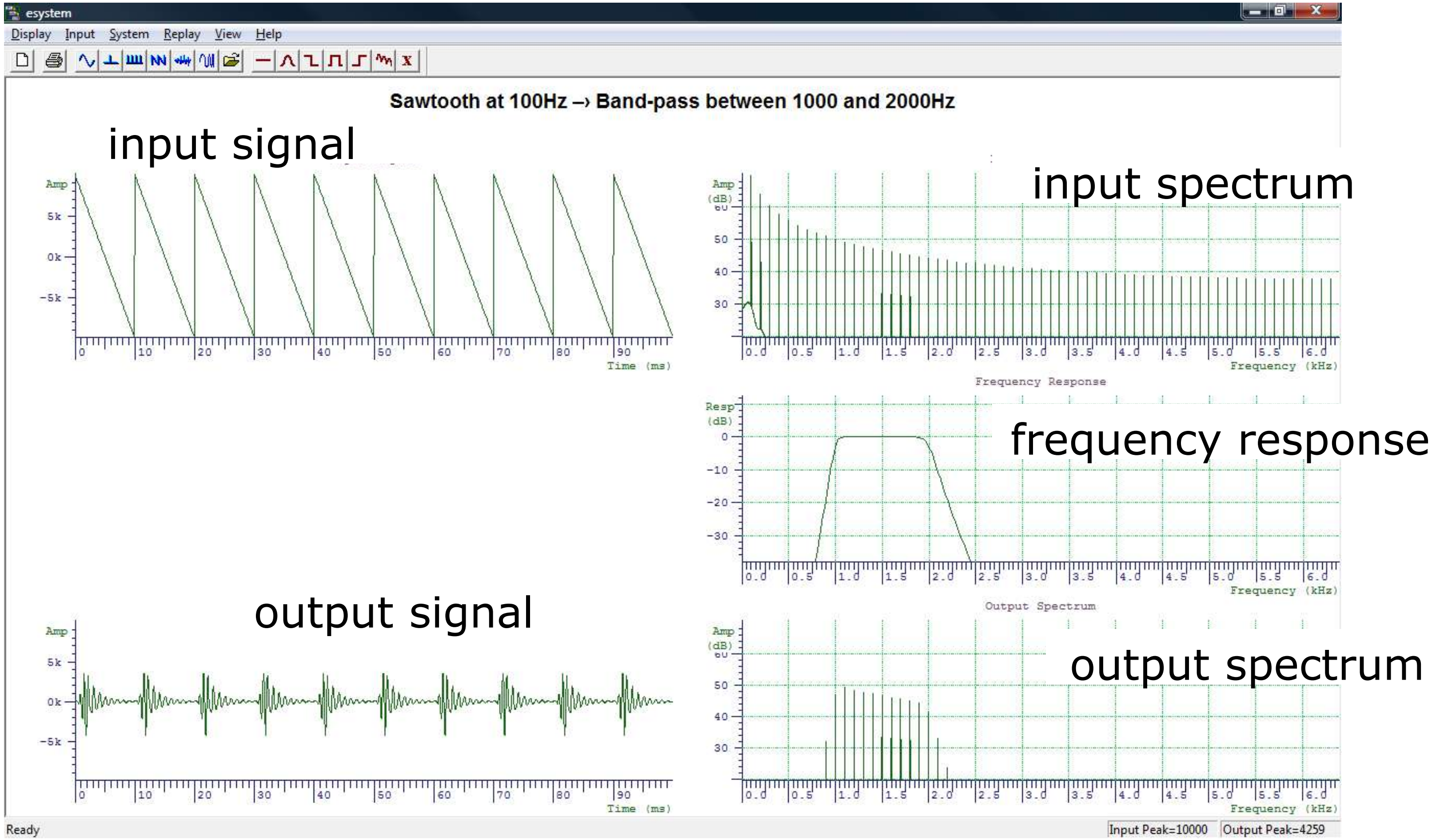
Using an aperiodic input (white noise): A *continuous* spectrum



Note that additions are done 'all at once'

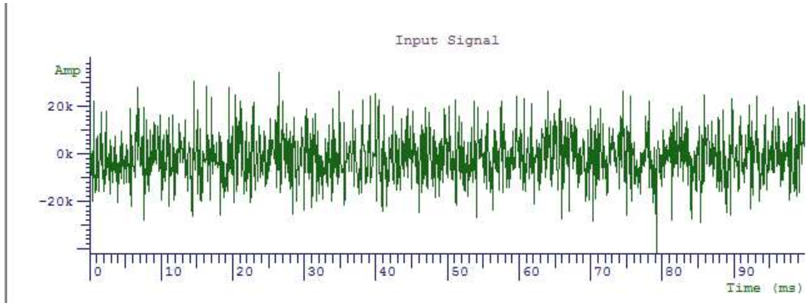
Consider this frequency response (what is it?)

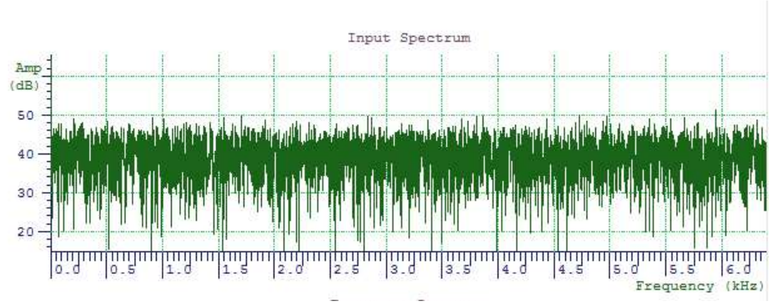




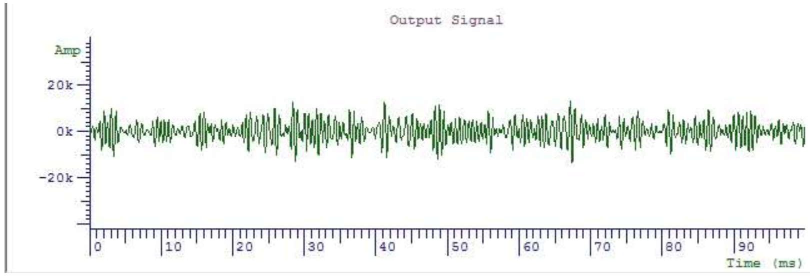
White Noise

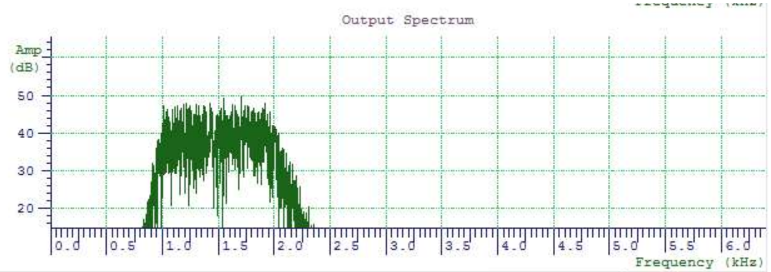
input





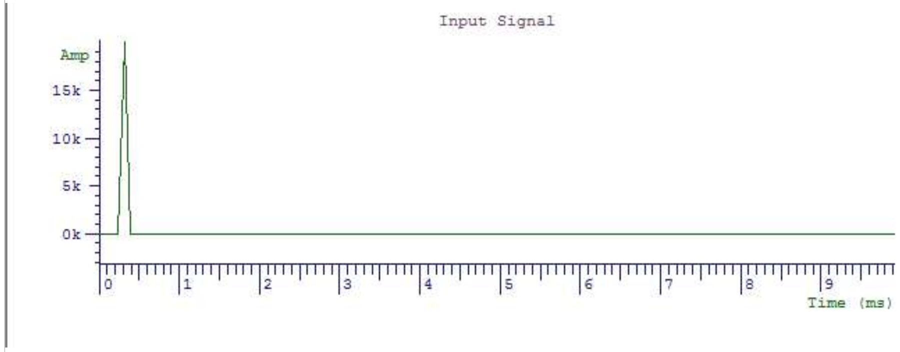
output

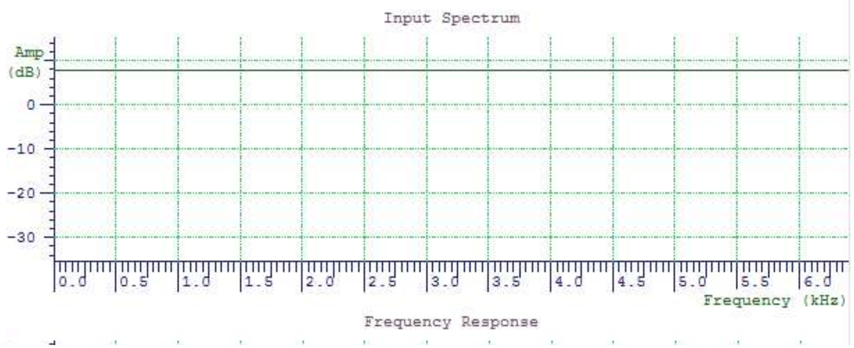




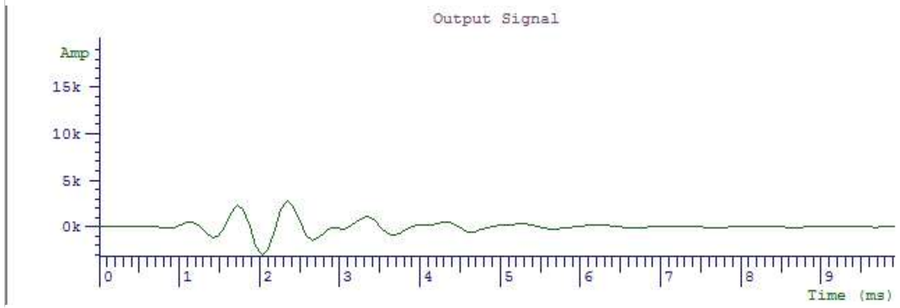
Single pulse

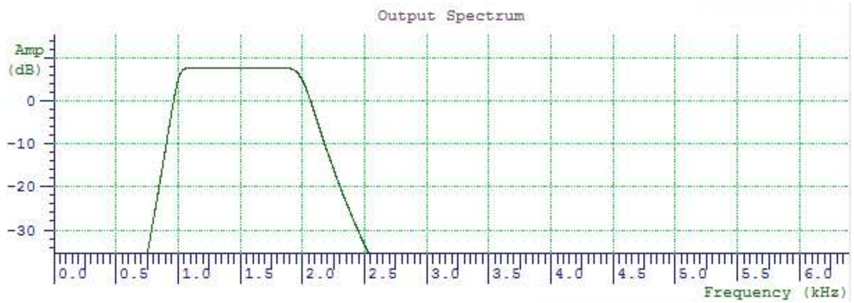
input



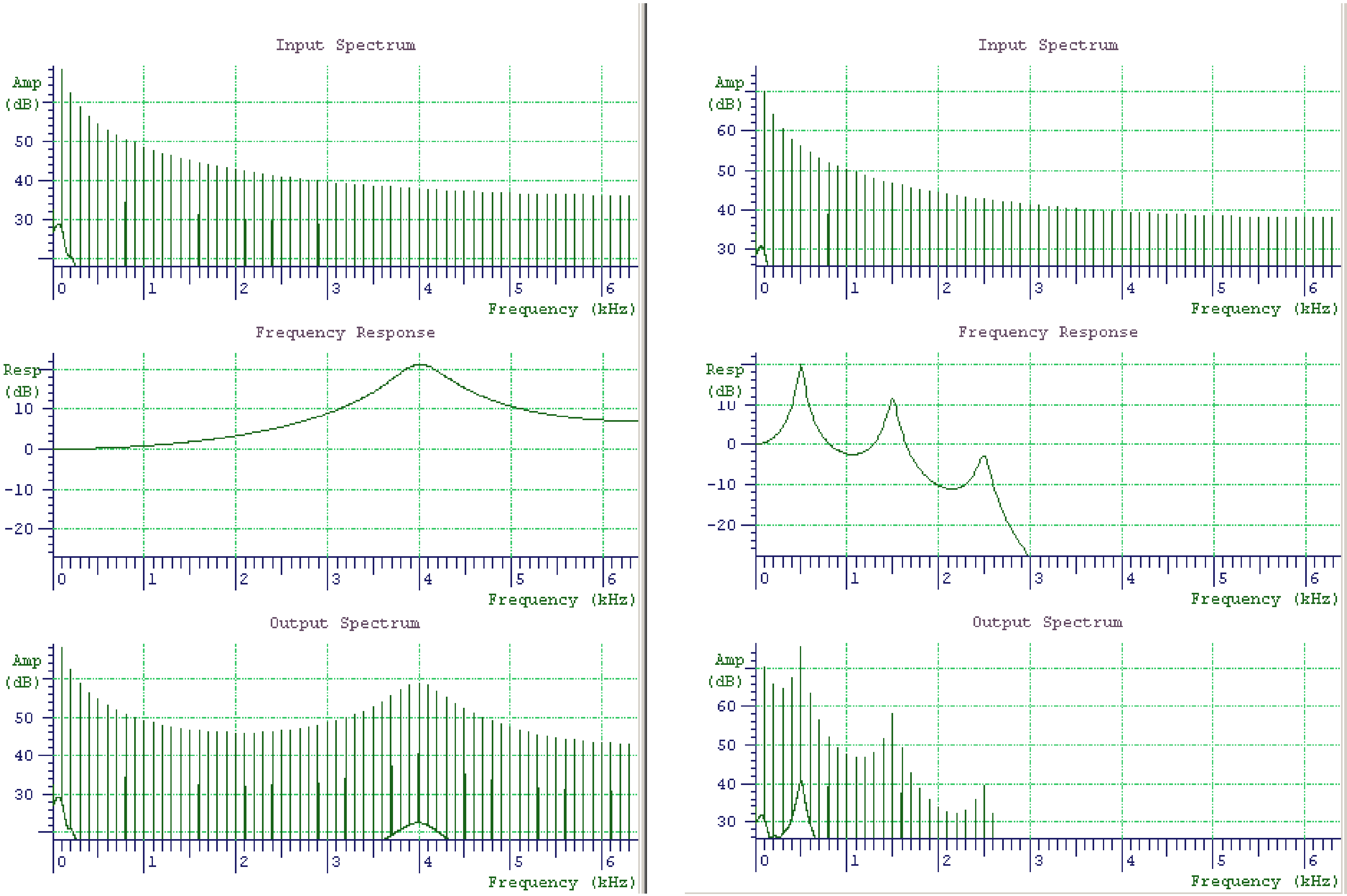


output





More complex examples



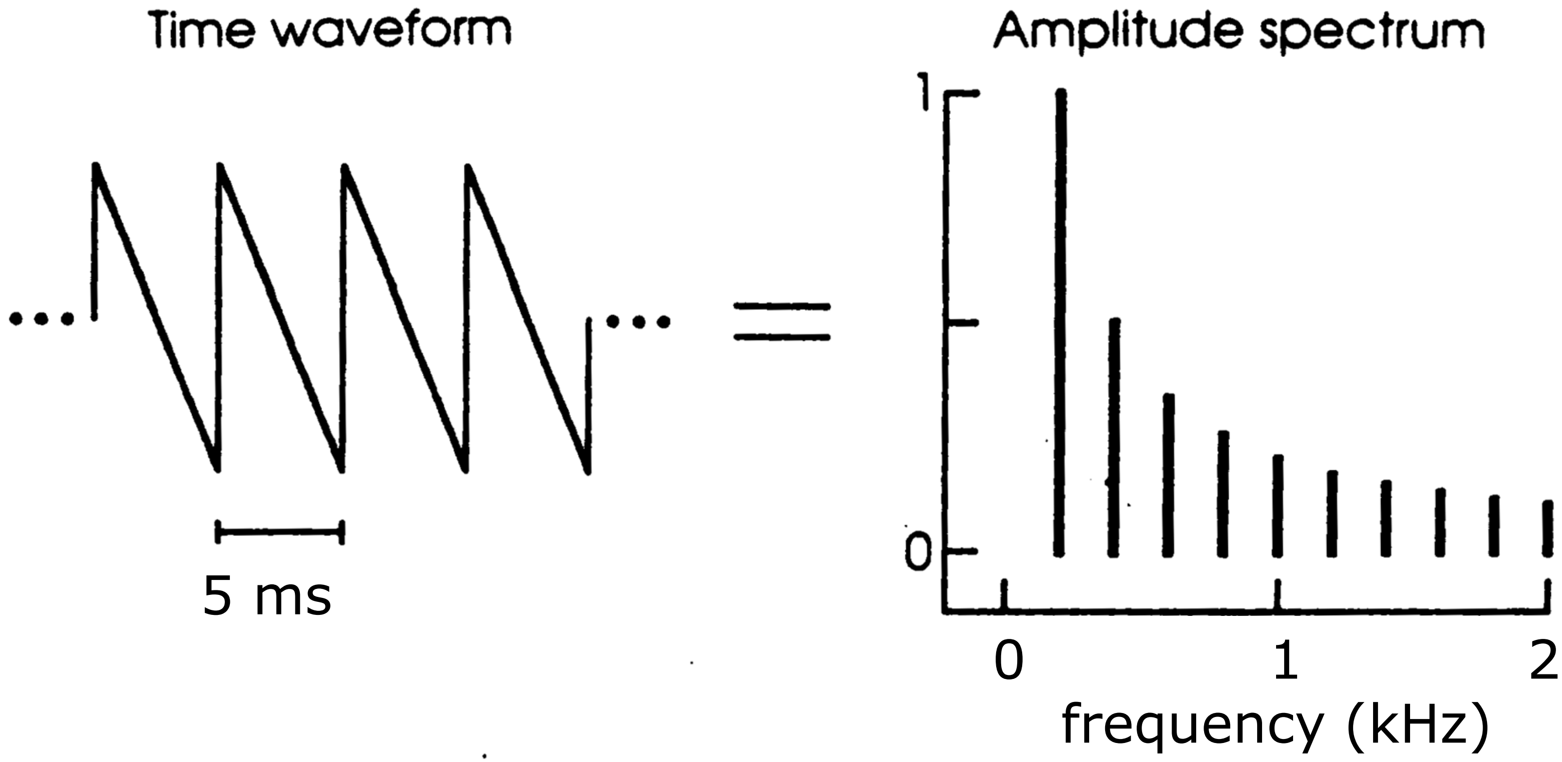
Bandpass filters & filterbanks

Practical spectral analysis

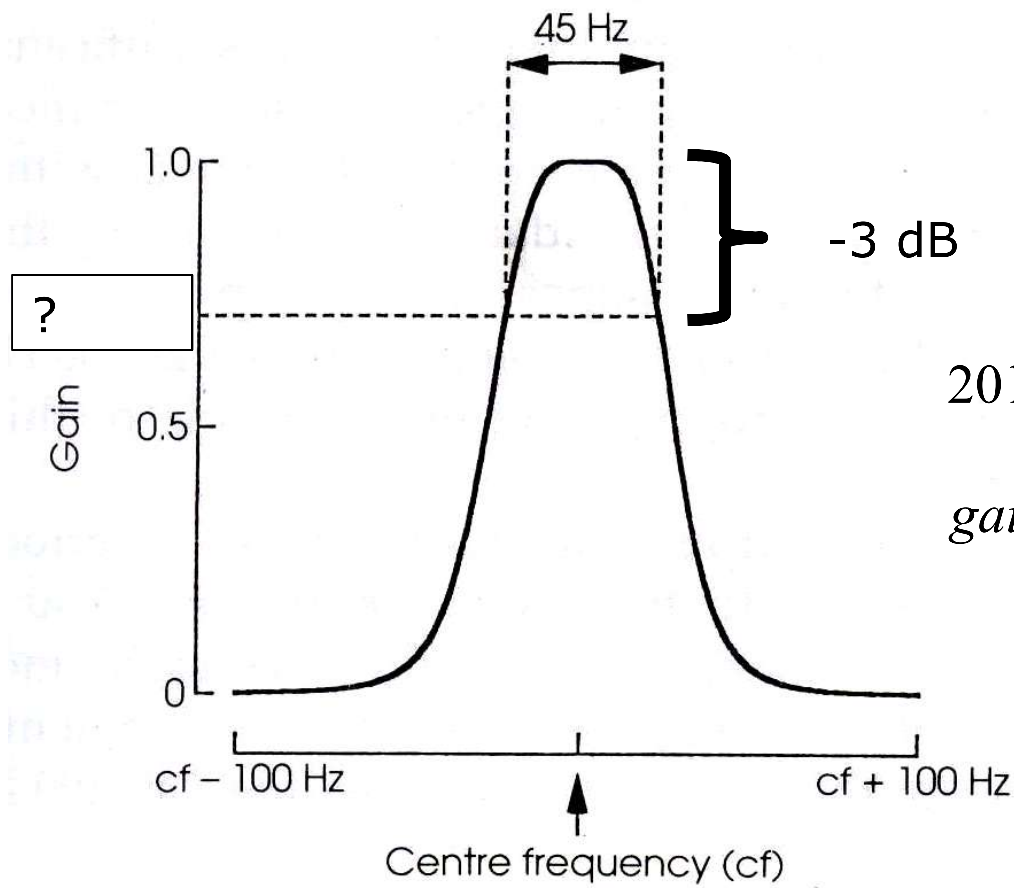
- Most analogue signals of interest are not easily mathematically specified …
  - so applying a Fourier transform directly (through an equation) is not possible
- Digital techniques allow the use of the FFT …
  - simply by sampling the waveform values
- How was this done back in the day? Or even now, in analogue form?
- What kind of LTI system separates out frequency components?

Try this out on an old friend …

Sawtooth amplitude spectrum



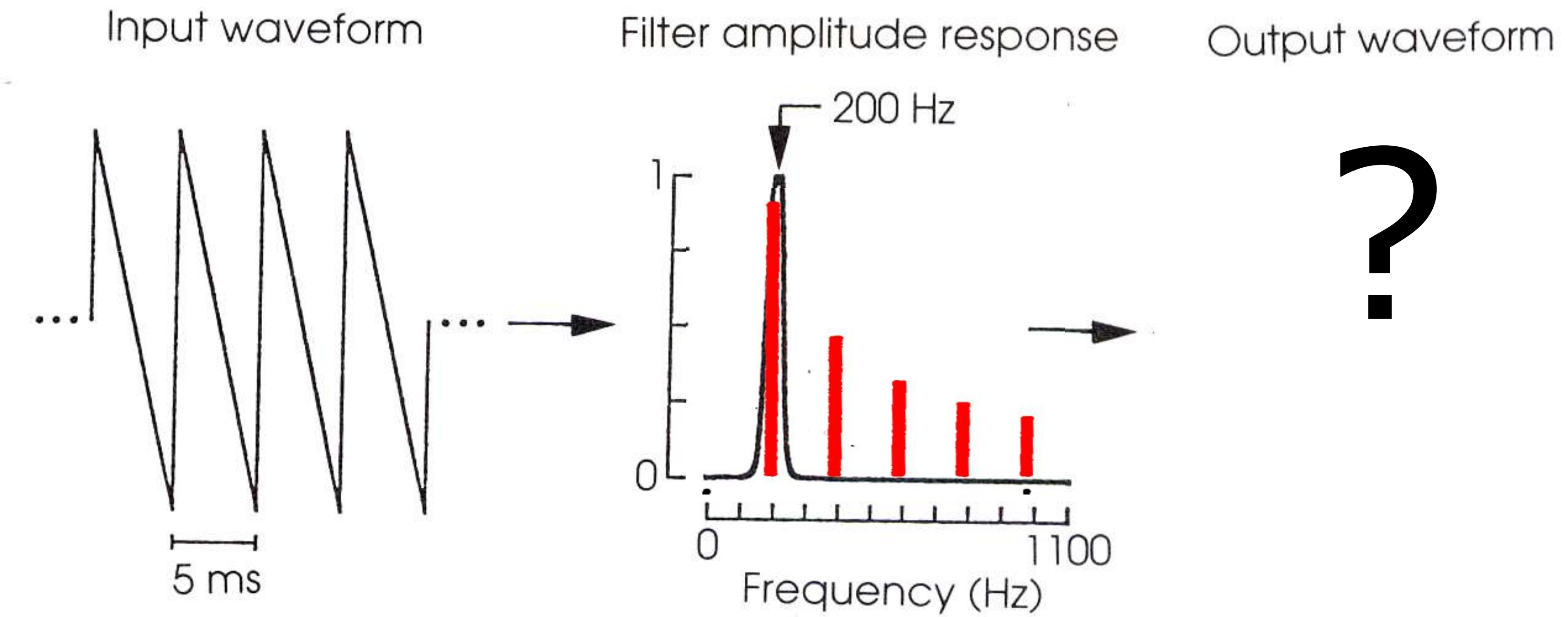
Need a bandpass filter with variable centre frequency



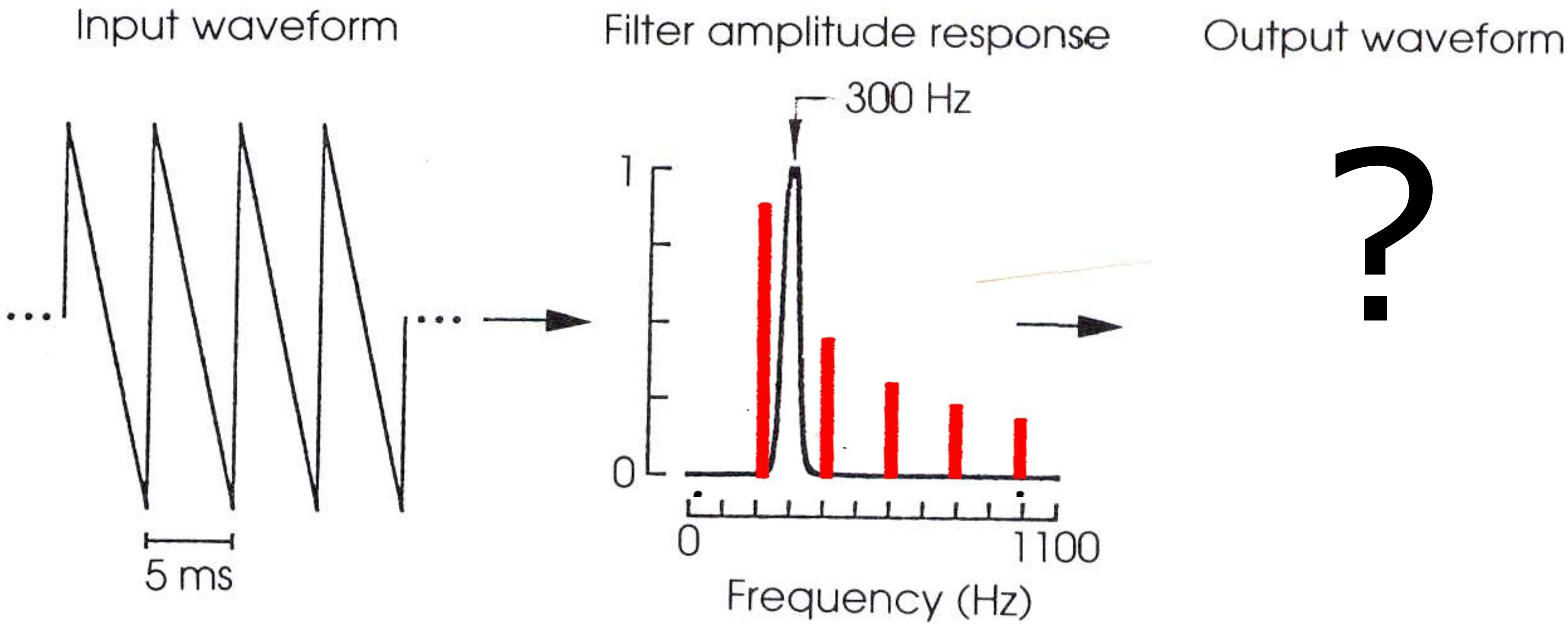
$$20\log(gain) = -3$$

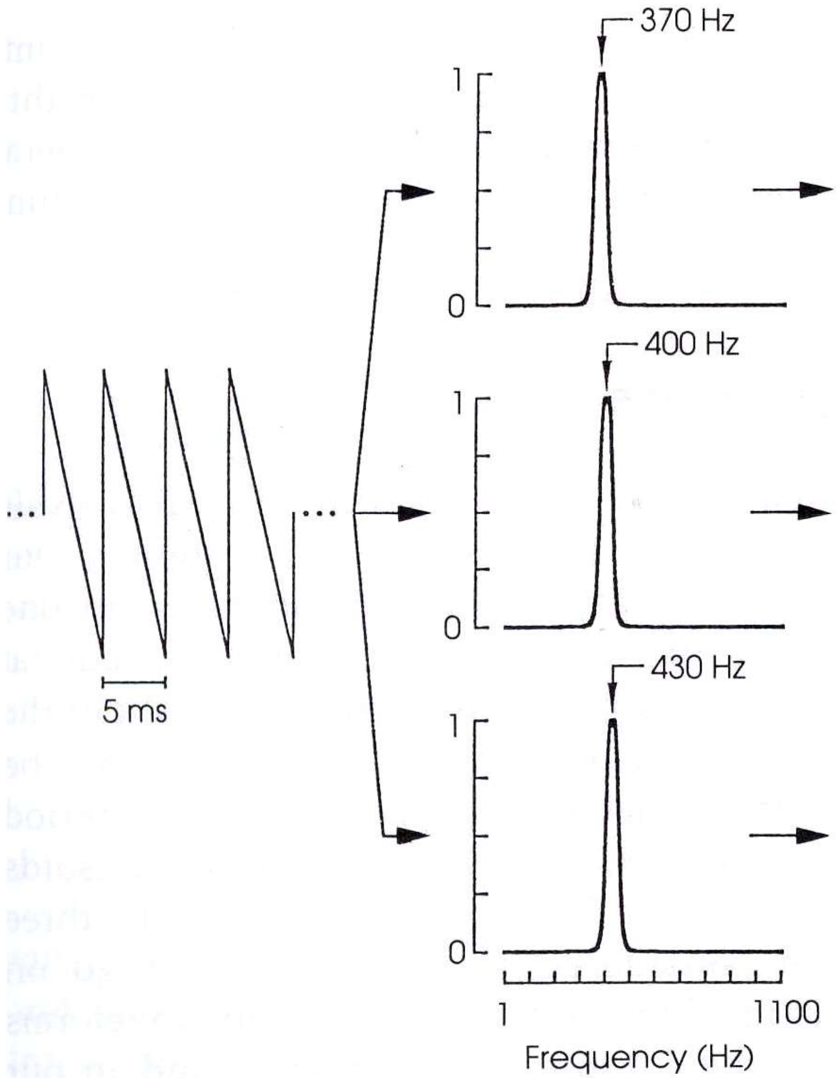
$$gain = 10^{-\frac{3}{20}} \approx 0.7$$

Tune filter to 200 Hz



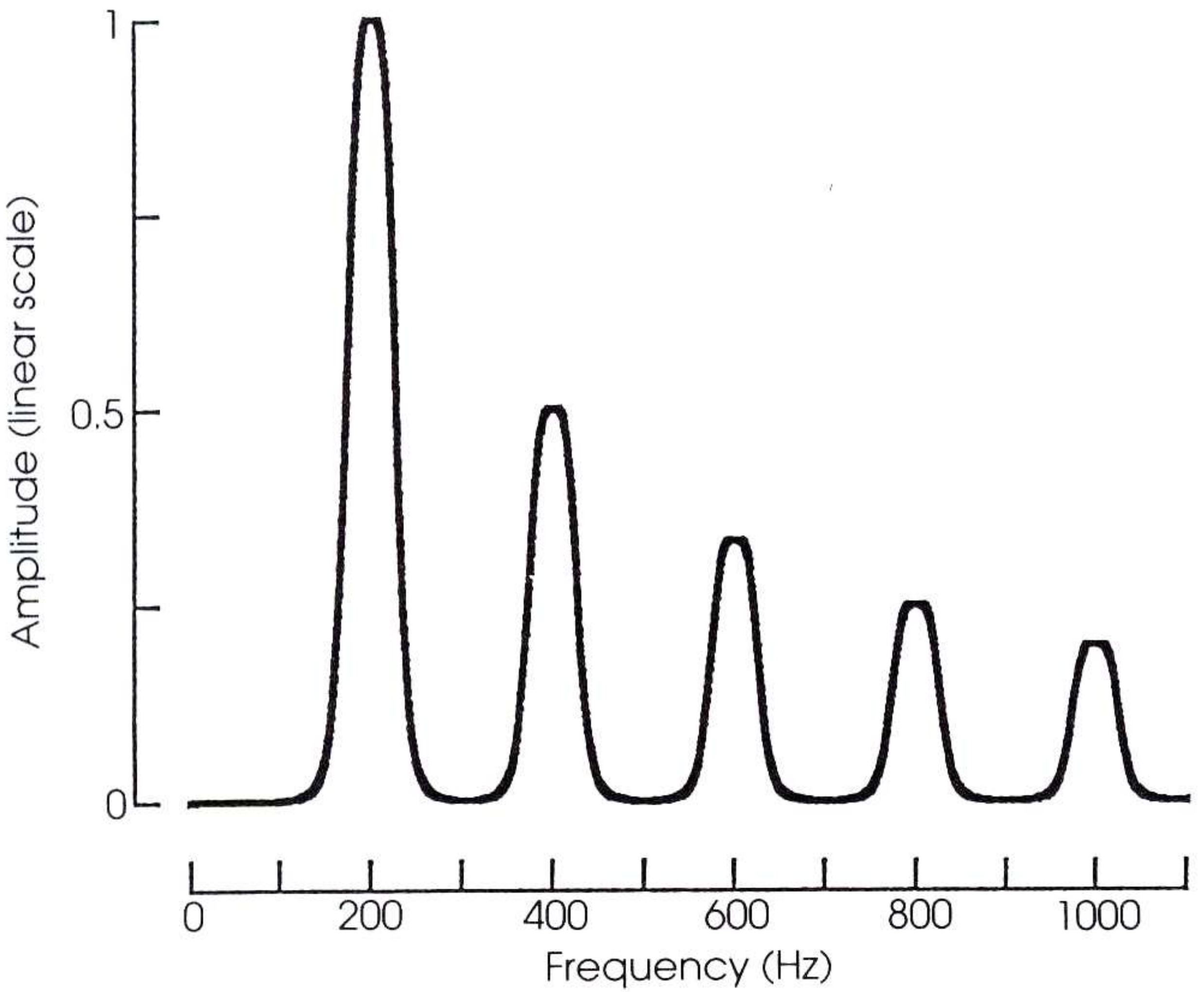
Tune filter to 300 Hz





Tune filter to inter-mediate frequen-cies

To construct the spectrum



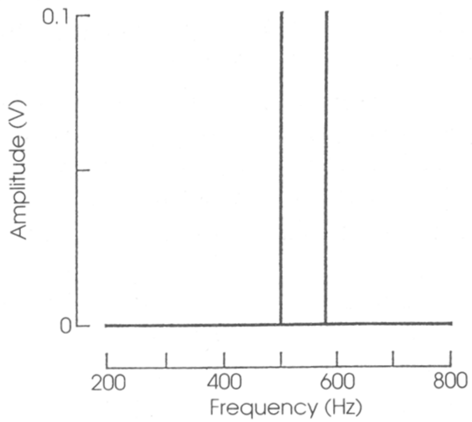
Can do this in two ways …

- As shown, with a tunable bandpass filter
  - cheap to implement, slow to run
- Or, with a *filter bank*
  - A set of bandpass filters whose centre frequencies are distributed over a desired frequency range
  - fast because of parallel processing but expensive in hardware
- Exotic fact you can ignore
  - an Fourier analysis can be thought of as implementing a filter bank

What filter properties affect the output of a filterbank?

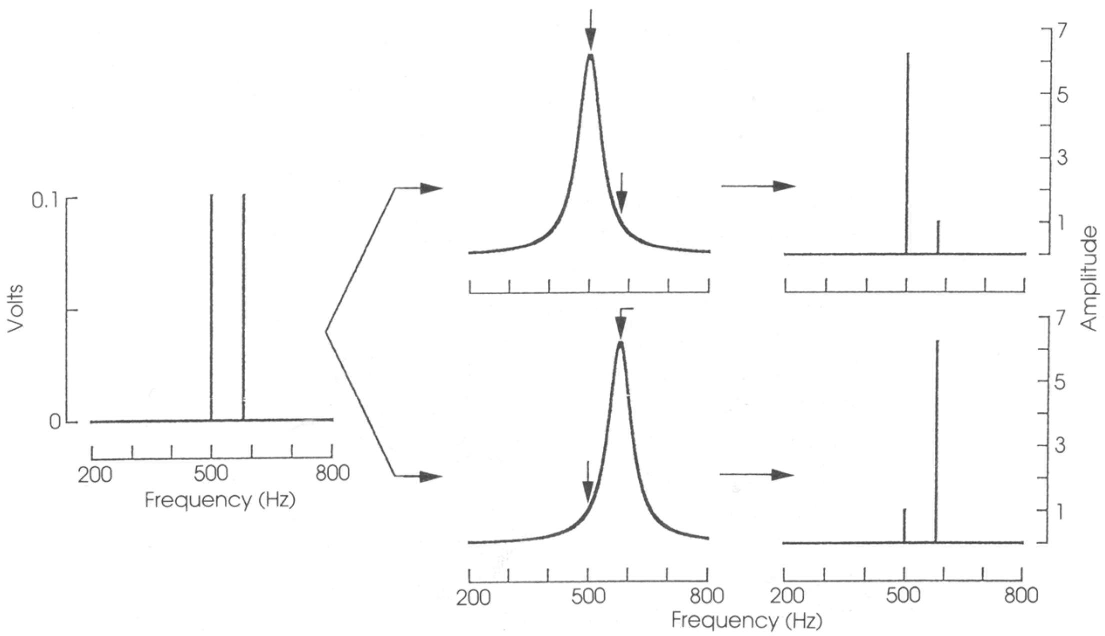
- ????
- ???? of filters in a filter bank determines the resolution of the spectrum
- Need to space filters relative to ????
- Why?
  - don't want holes in the spectrum
  - could miss spectral components

How the properties of a filter bank influence signals through it: I. Resolution in frequency

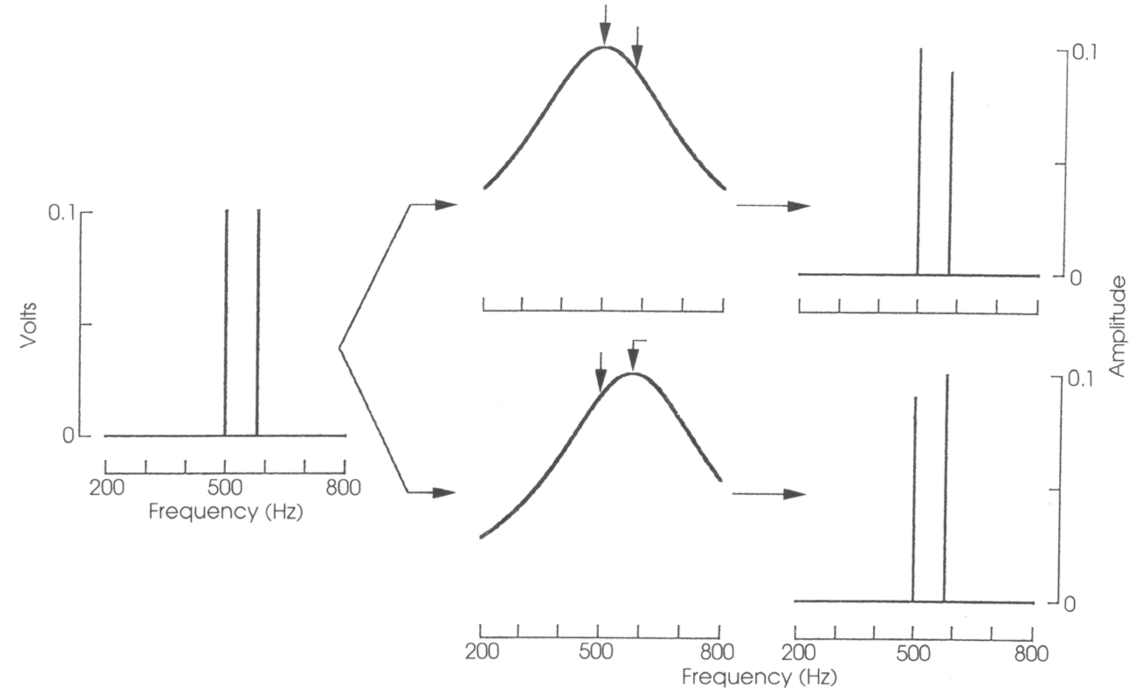


Consider a signal that consists of two sinusoids reasonably close in frequency, which are to be analysed in a filter bank.

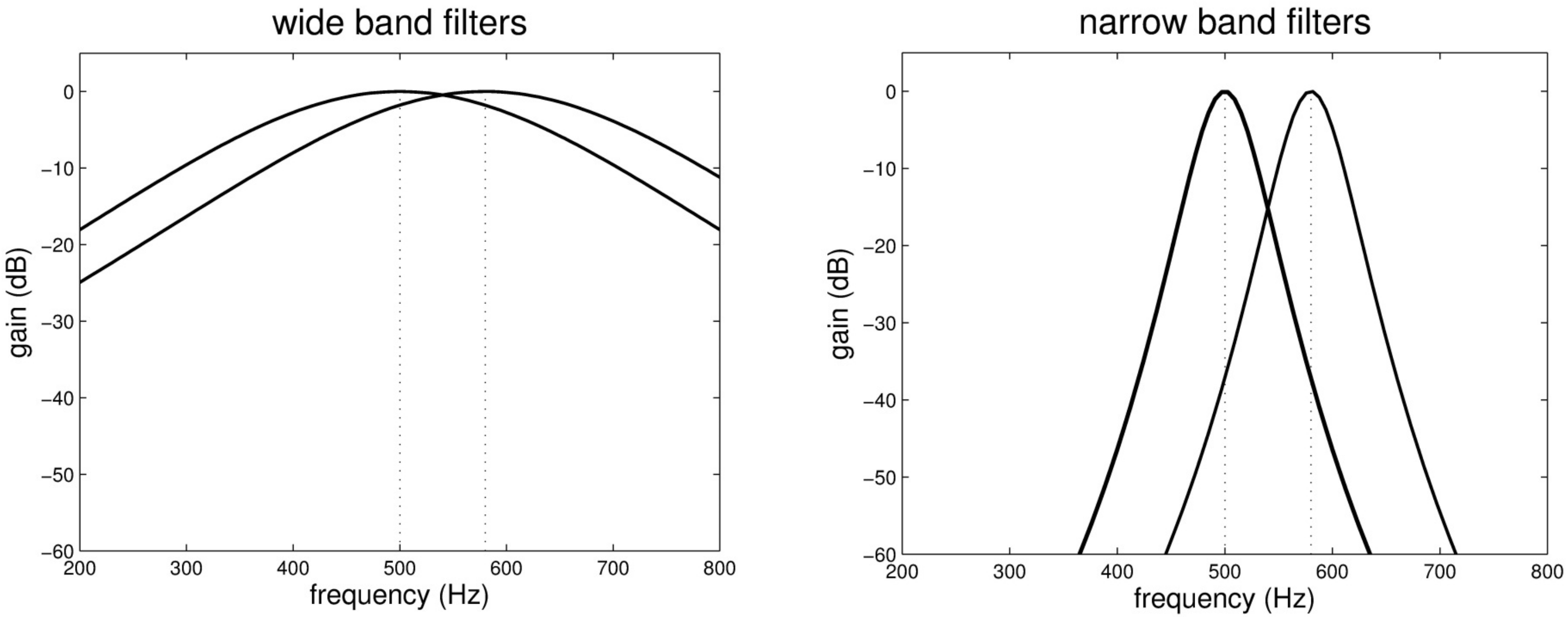
Filtering through narrow filters



Filtering through wide filters

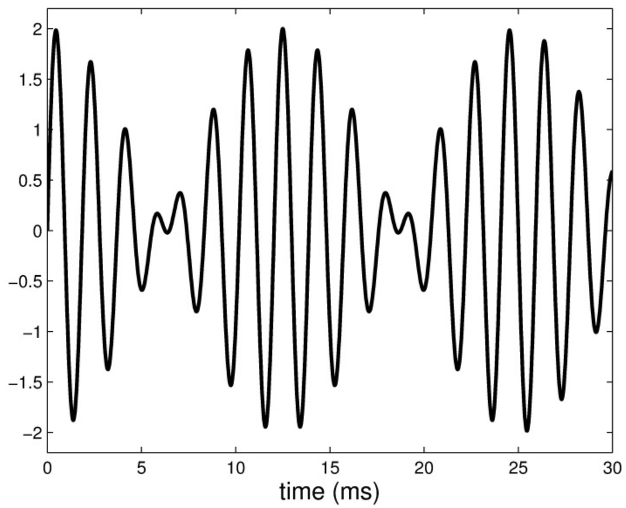


A more extreme example

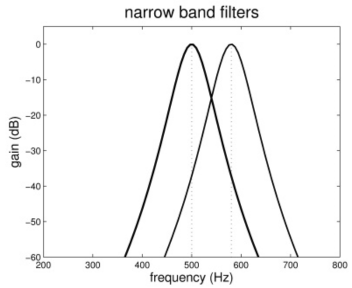


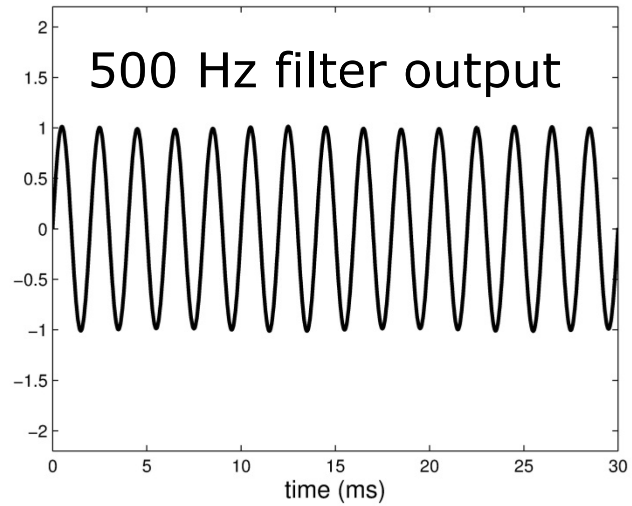
Narrow band filters

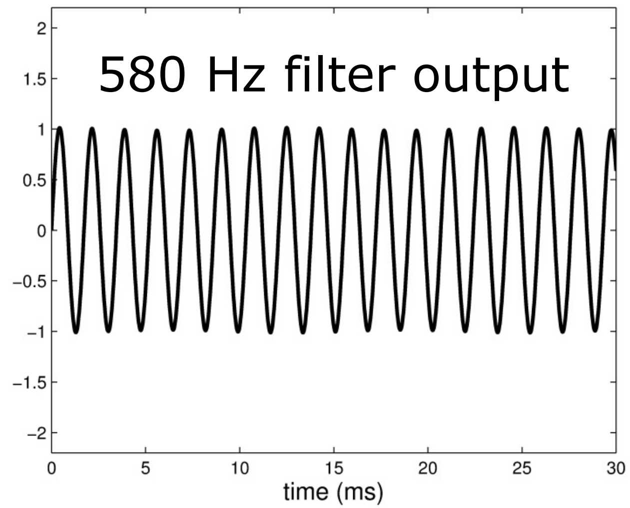
input wave



500 Hz + 580 Hz

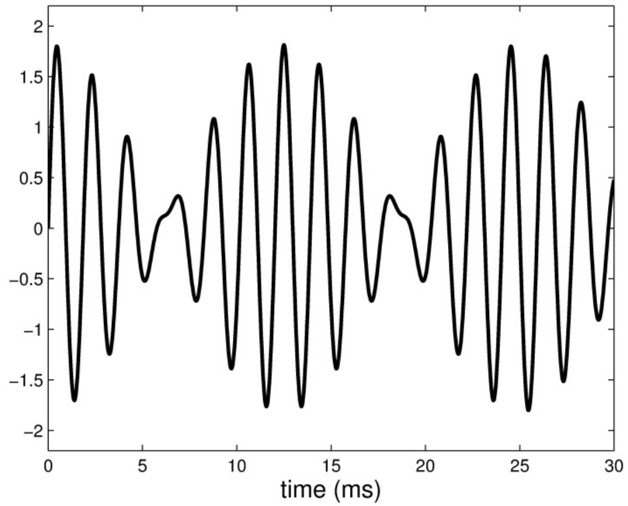




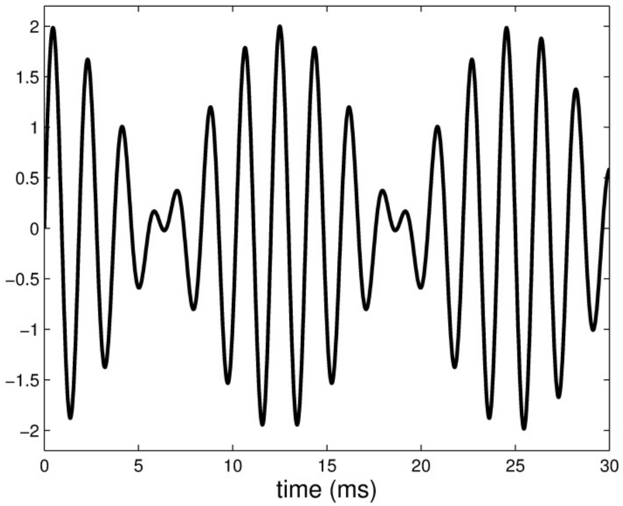


Wide band filters

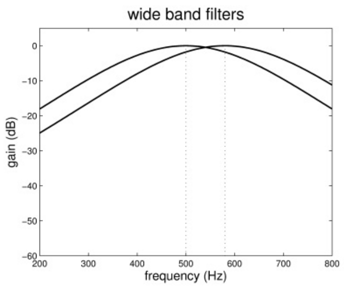
500 Hz filter output

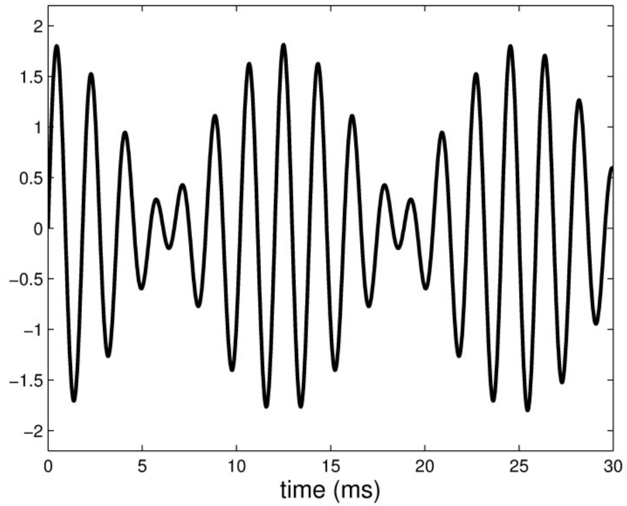


input wave

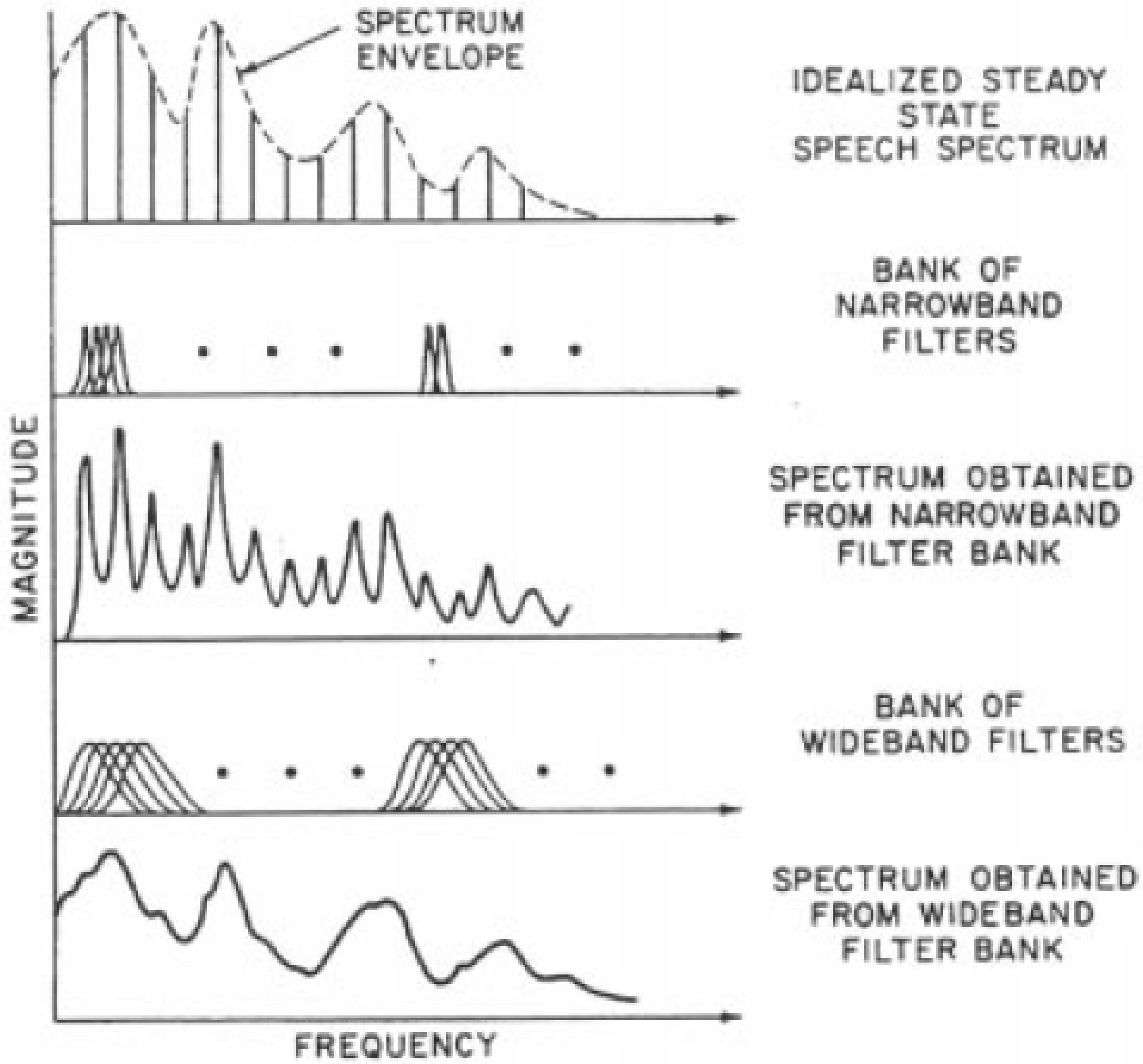


500 Hz + 580 Hz





580 Hz filter output

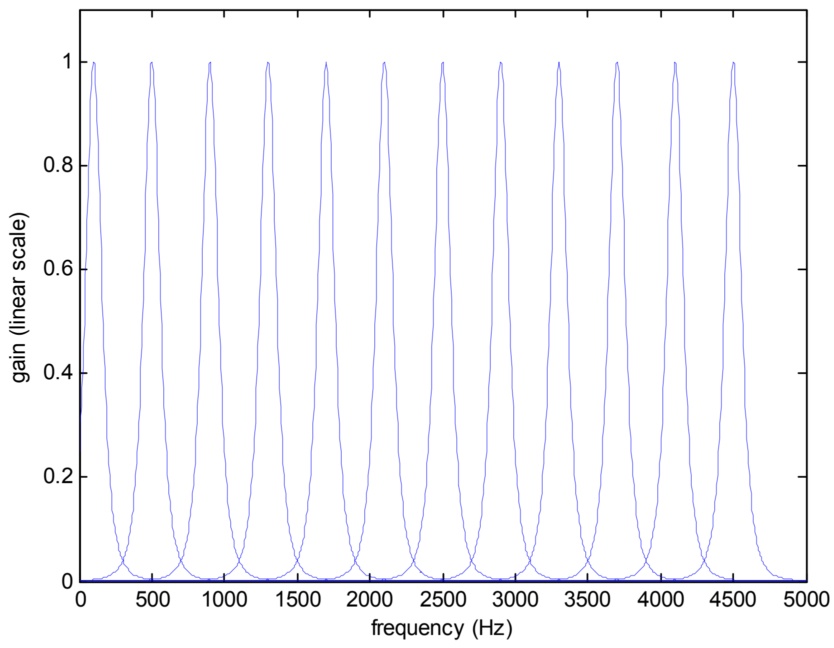


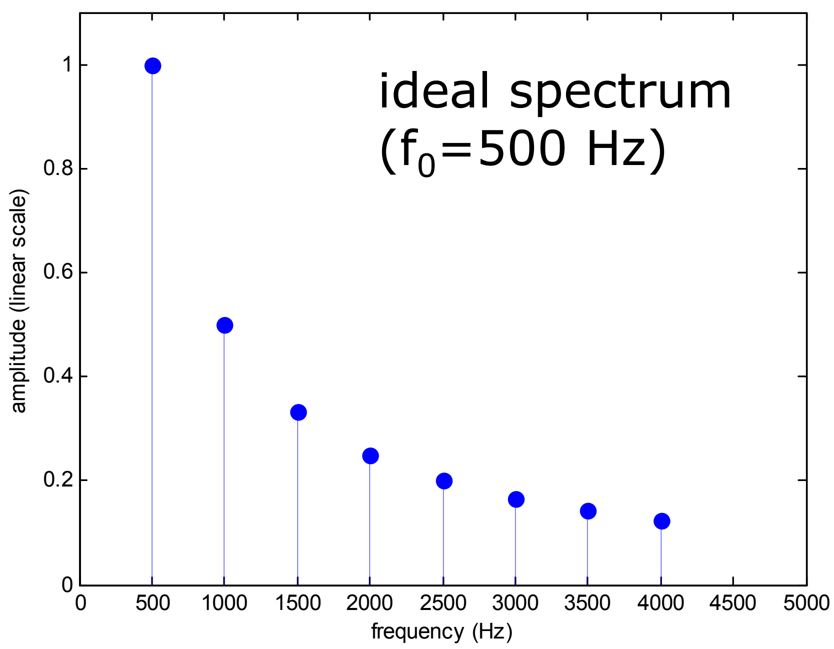
COMPARISON OF (Idealized) MEASURED SPECTRA FOR WIDE AND NARROW FILTER BANK ANALYZERS

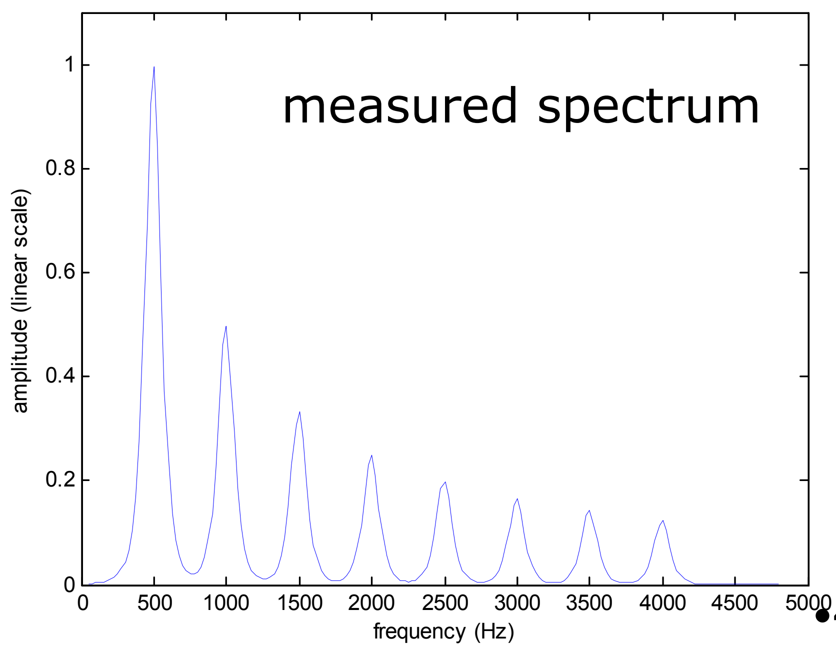
Spectral analysis with a filter-bank:

No single unique spectrum!

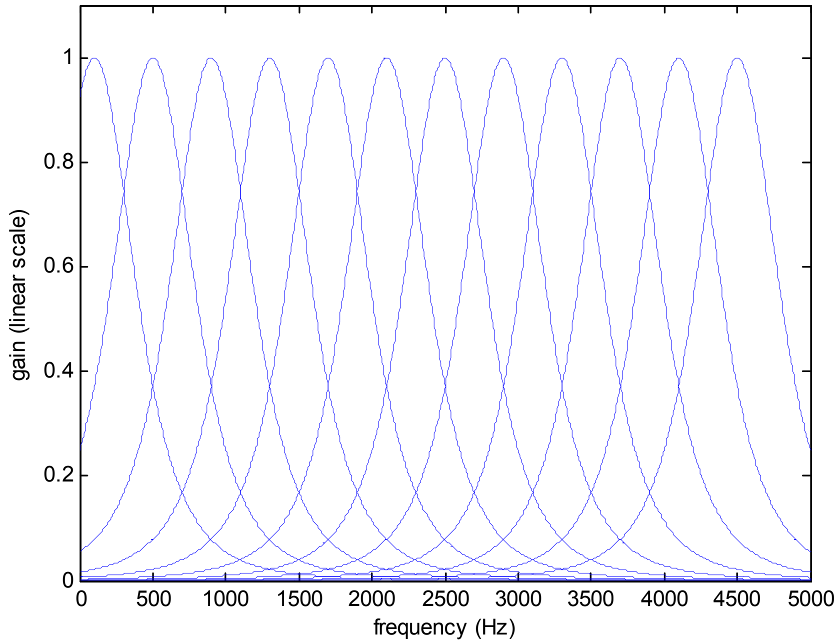
Example filter bank and analysis (bandwidth ≈ 100 Hz)

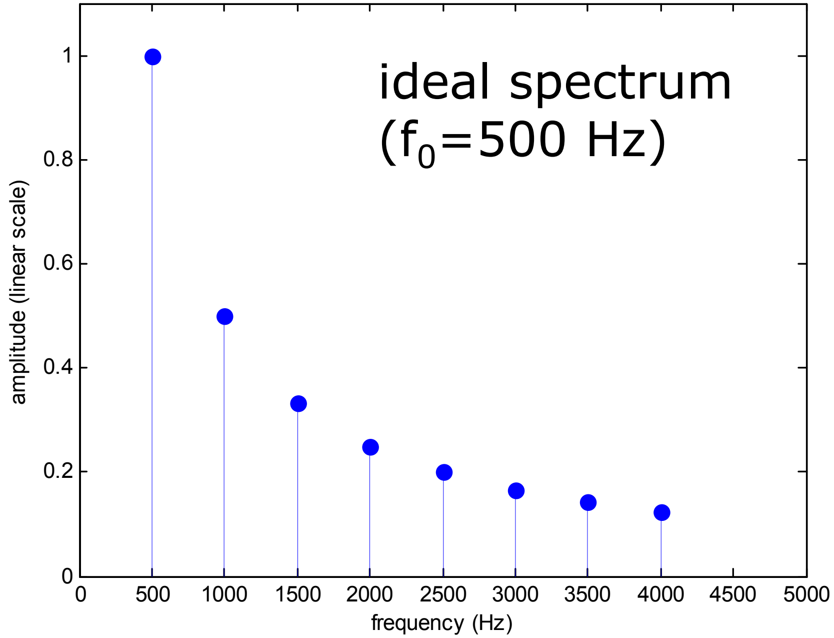


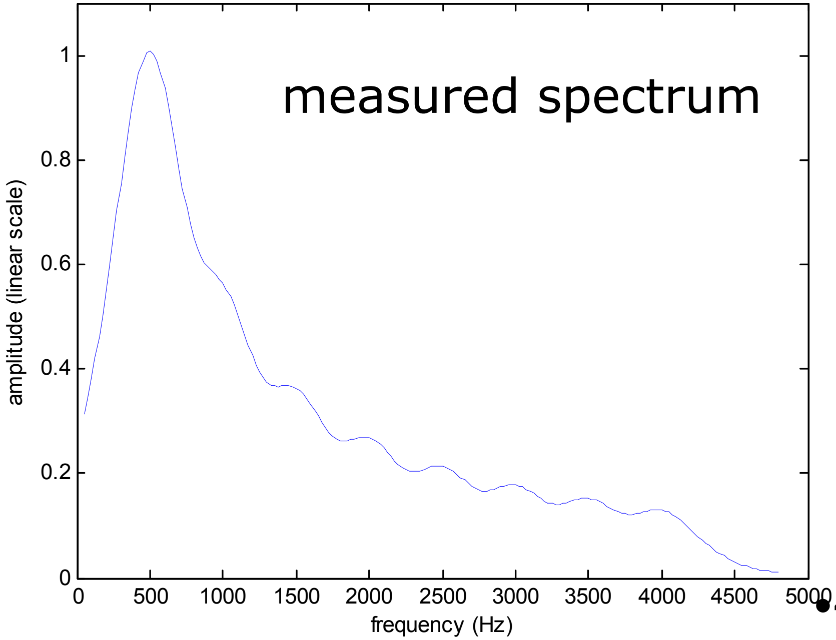




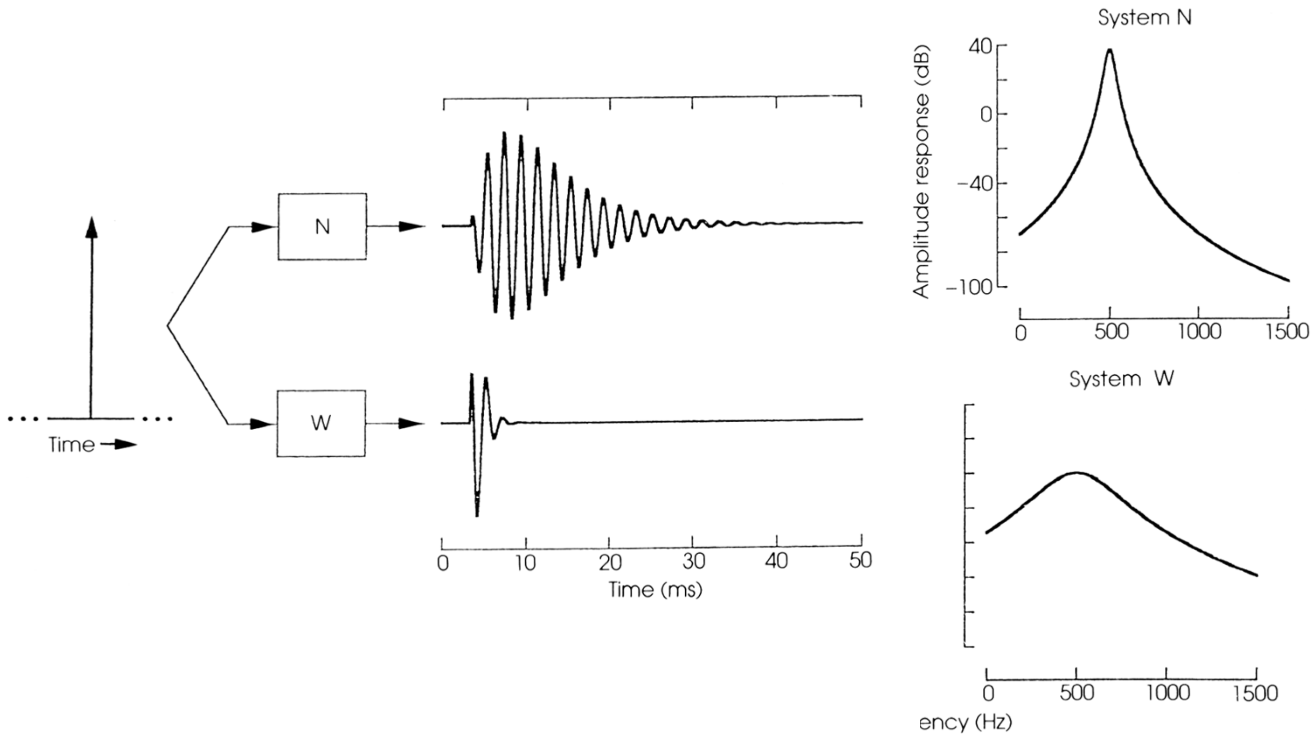
Example filter bank and analysis (bandwidth ≈ 500 Hz)





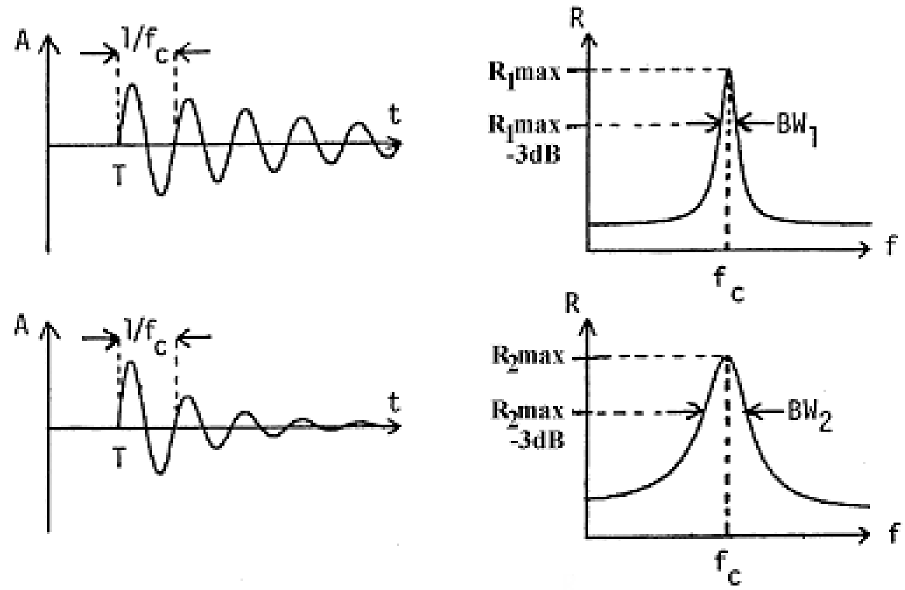


Impulses through narrow and wide filters



Bandwidth & Damping

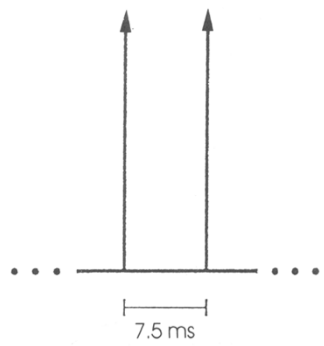
- Two ways of describing the same thing:
  - **Narrow** Bandwidth = **Low** Damping
  - **Wide** Bandwidth = **High** Damping



Summary

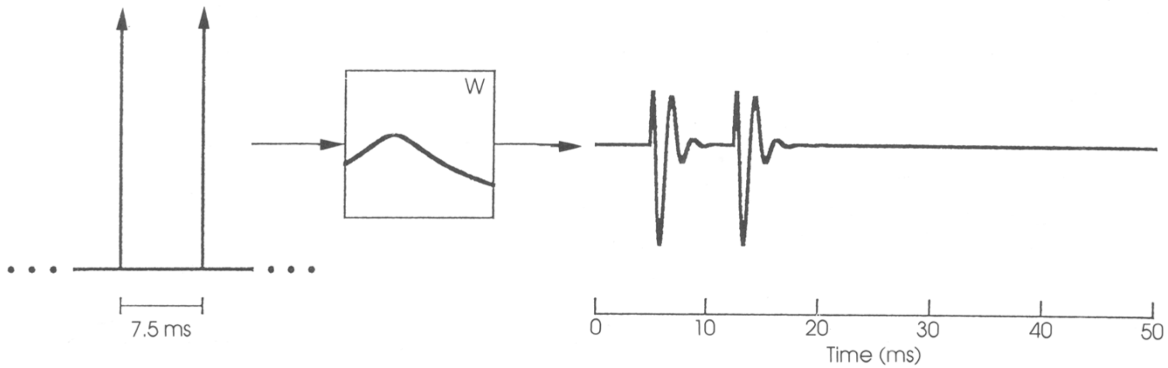
- Bandpass filters with a long impulse response have narrow frequency responses.
- Bandpass filters with a short impulse response have broad frequency responses.

How the properties of a filter bank influence signals through it: II. Resolution in time

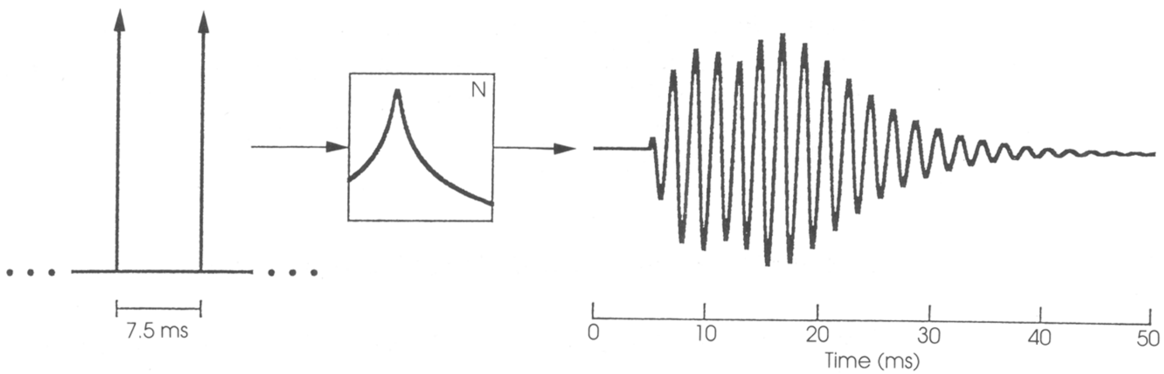


Consider a signal that consists of two impulses reasonably close in time, which are to be analysed in a filter bank.

Filtering through a wide filter



Filtering through a narrow filter

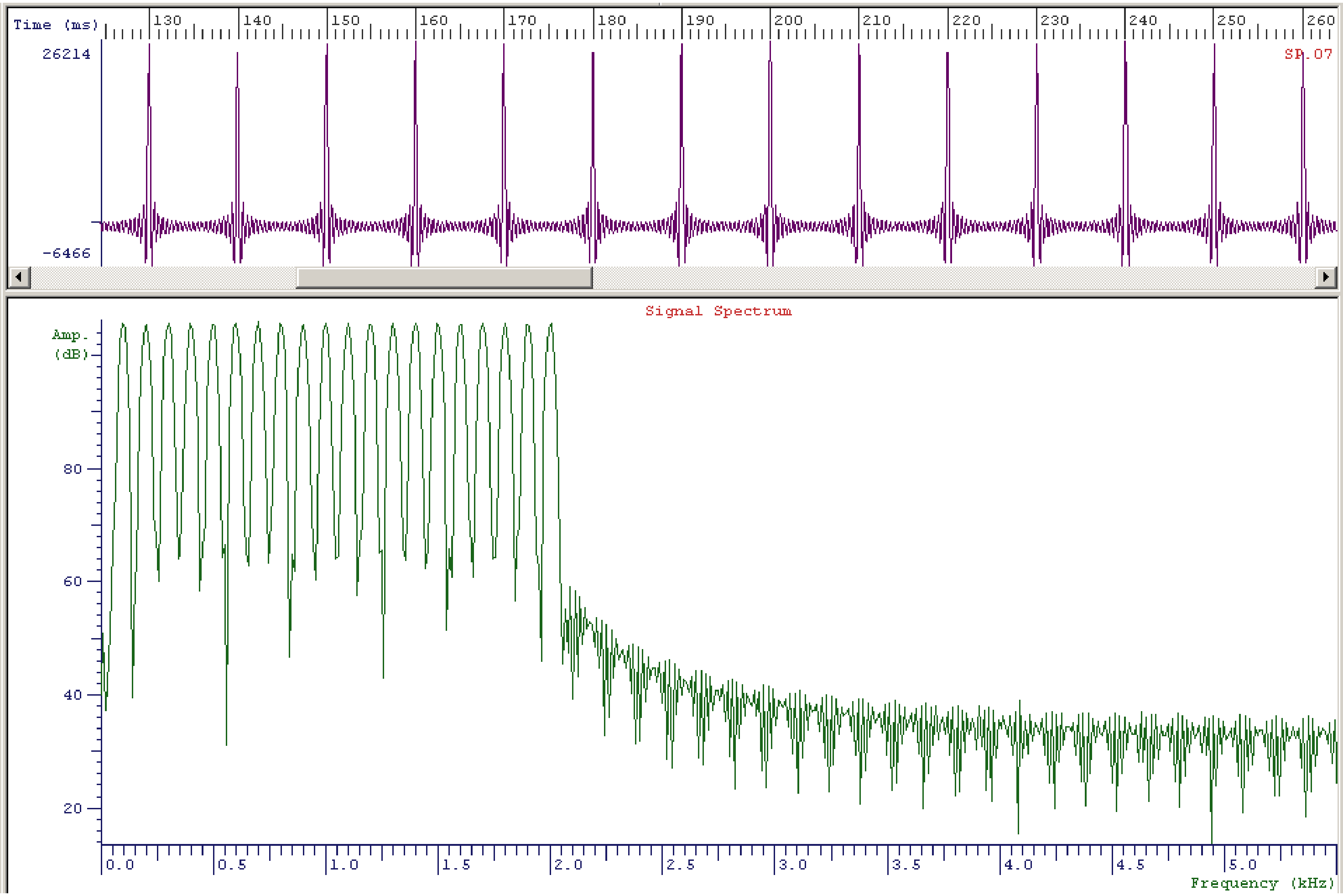


Summary

- Filter banks which consist of relatively narrow filters are good for seeing fine spectral detail …
  - but poor for temporal detail
- Filter banks which consist of relatively wide filters are good for seeing fine temporal detail …
  - but poor for spectral detail

Applying these concepts to a complex periodic wave consisting of 20 equal-amplitude harmonics of 100 Hz

A complex periodic wave consisting of 20 equal-amplitude harmonics of 100 Hz

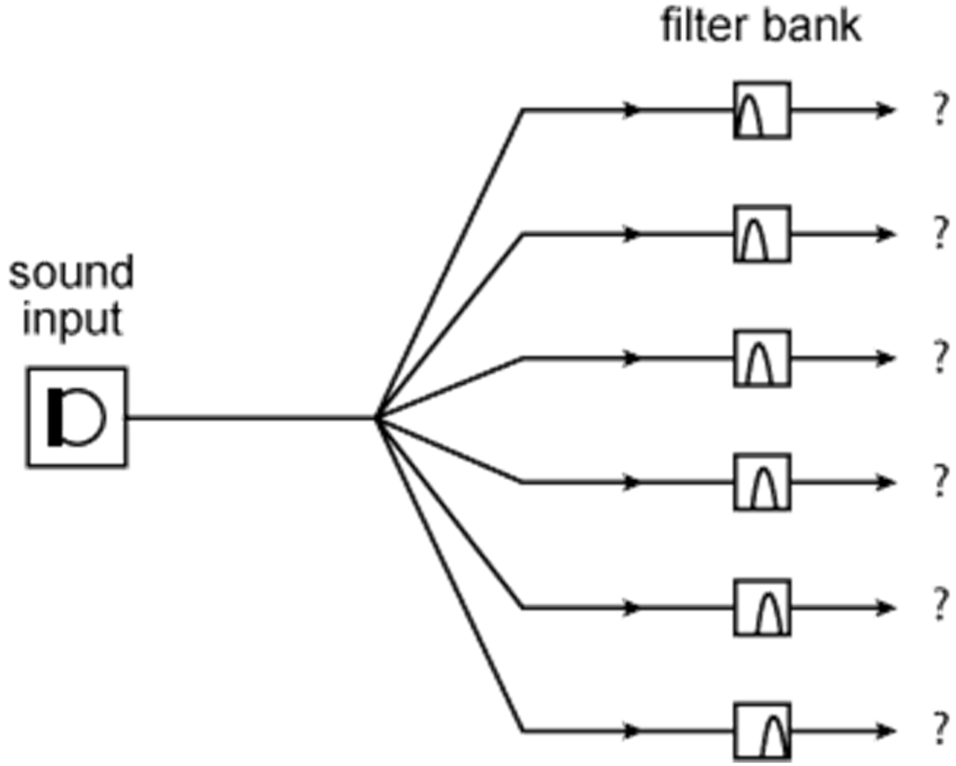


Narrow-band (50 Hz) filtering at 200, 250, 300, 350 and 400 Hz

Wide-band (300 Hz) filtering at 200, 250, 300, 350 and 400 Hz

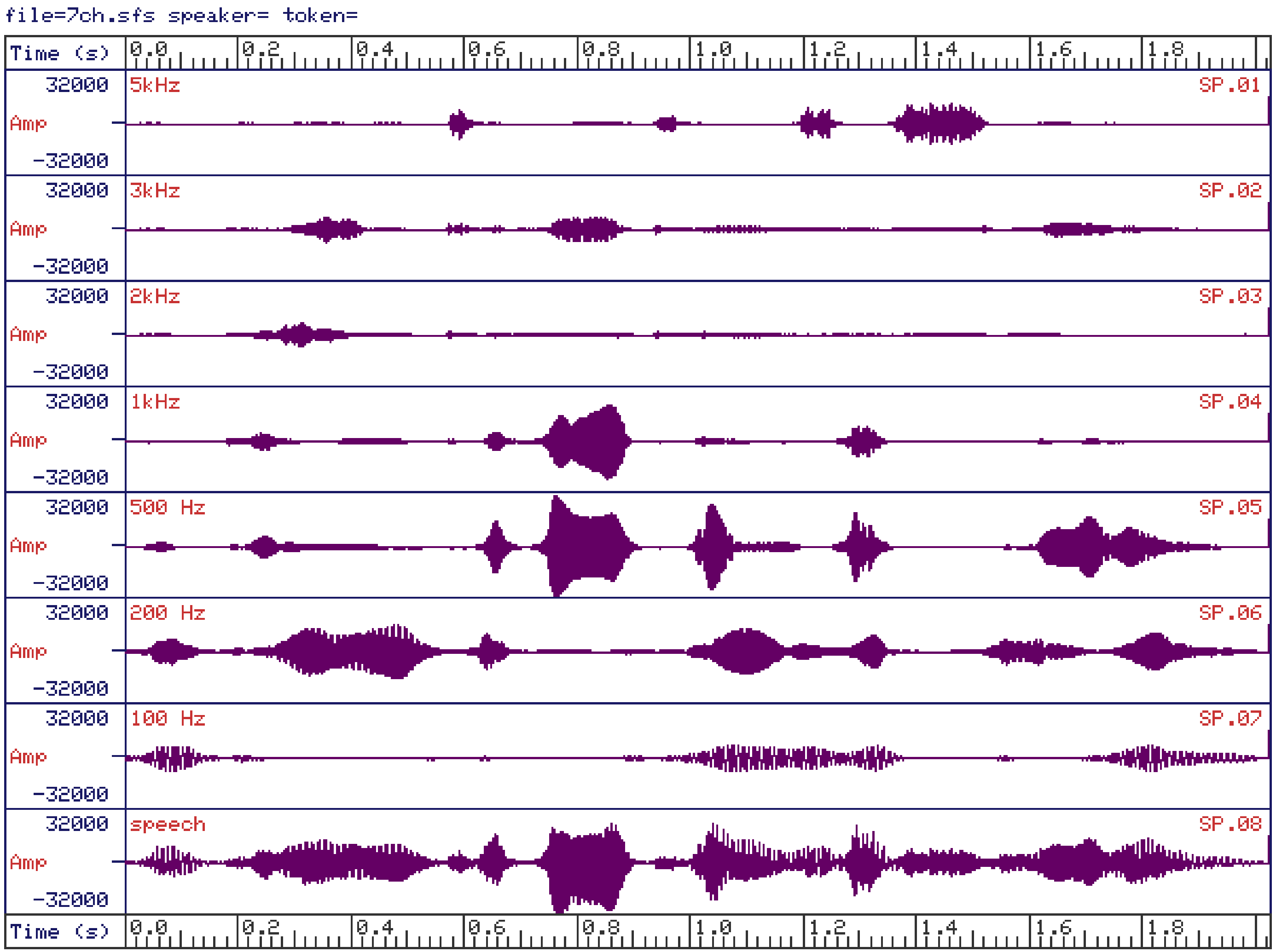
what do you see?

What does a filter bank do to a speech waveform?

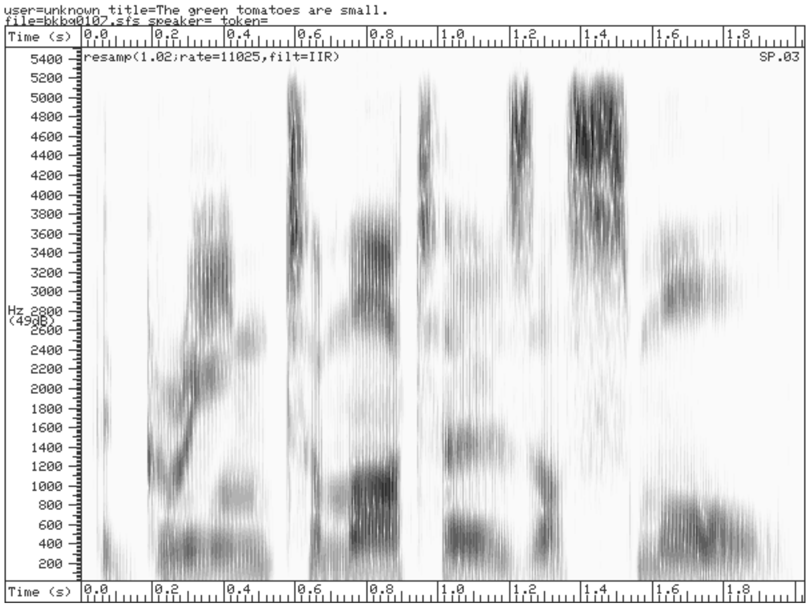


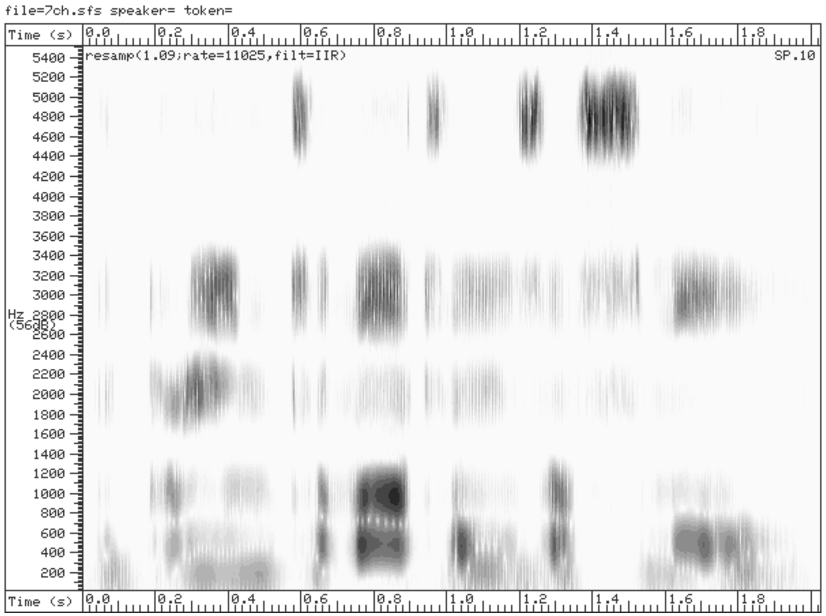
a 6-channel filter bank

Narrow bands of speech at different frequencies: Individual outputs from a filter bank

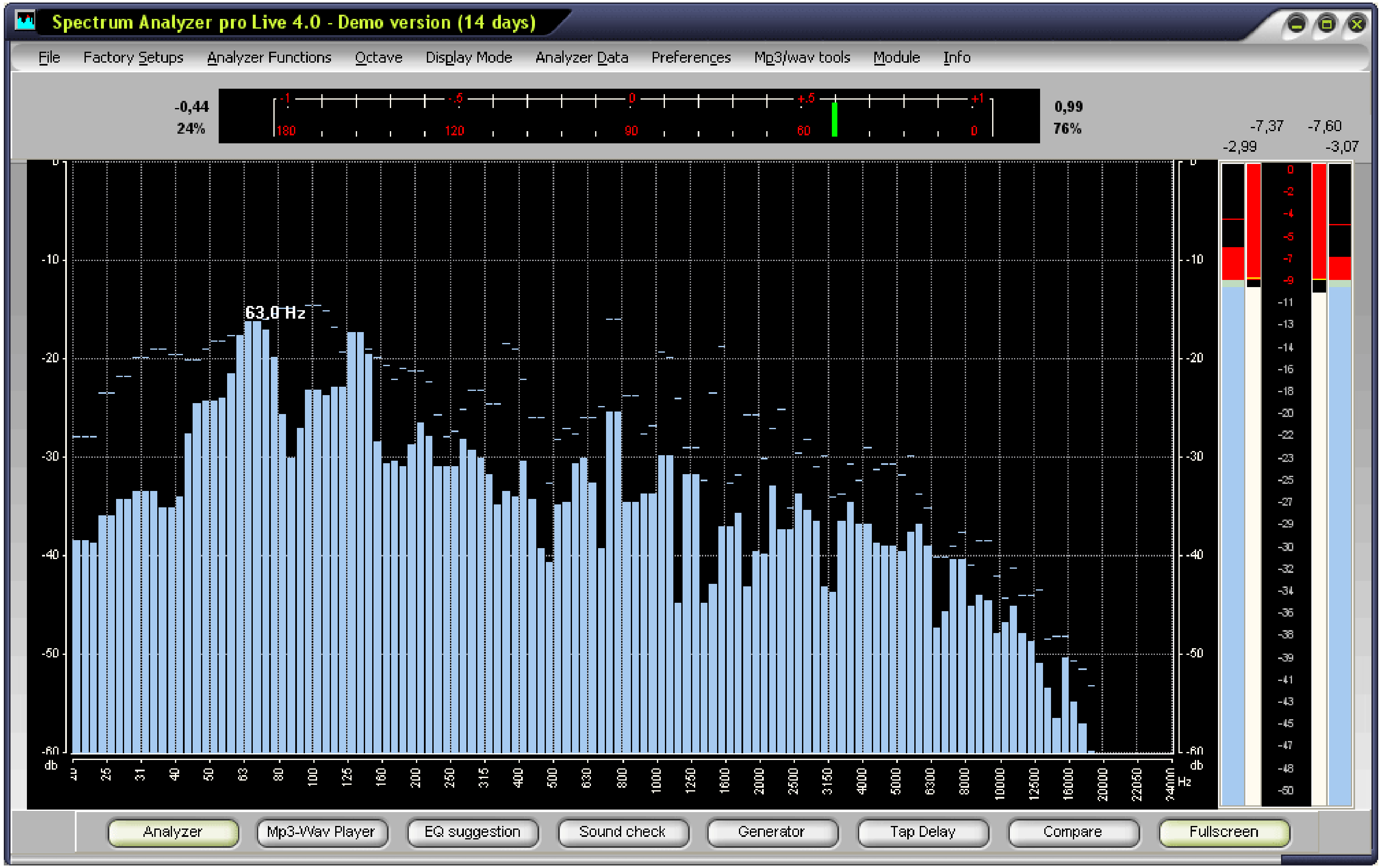


Of course, you need many more filters in the filter bank than seven.



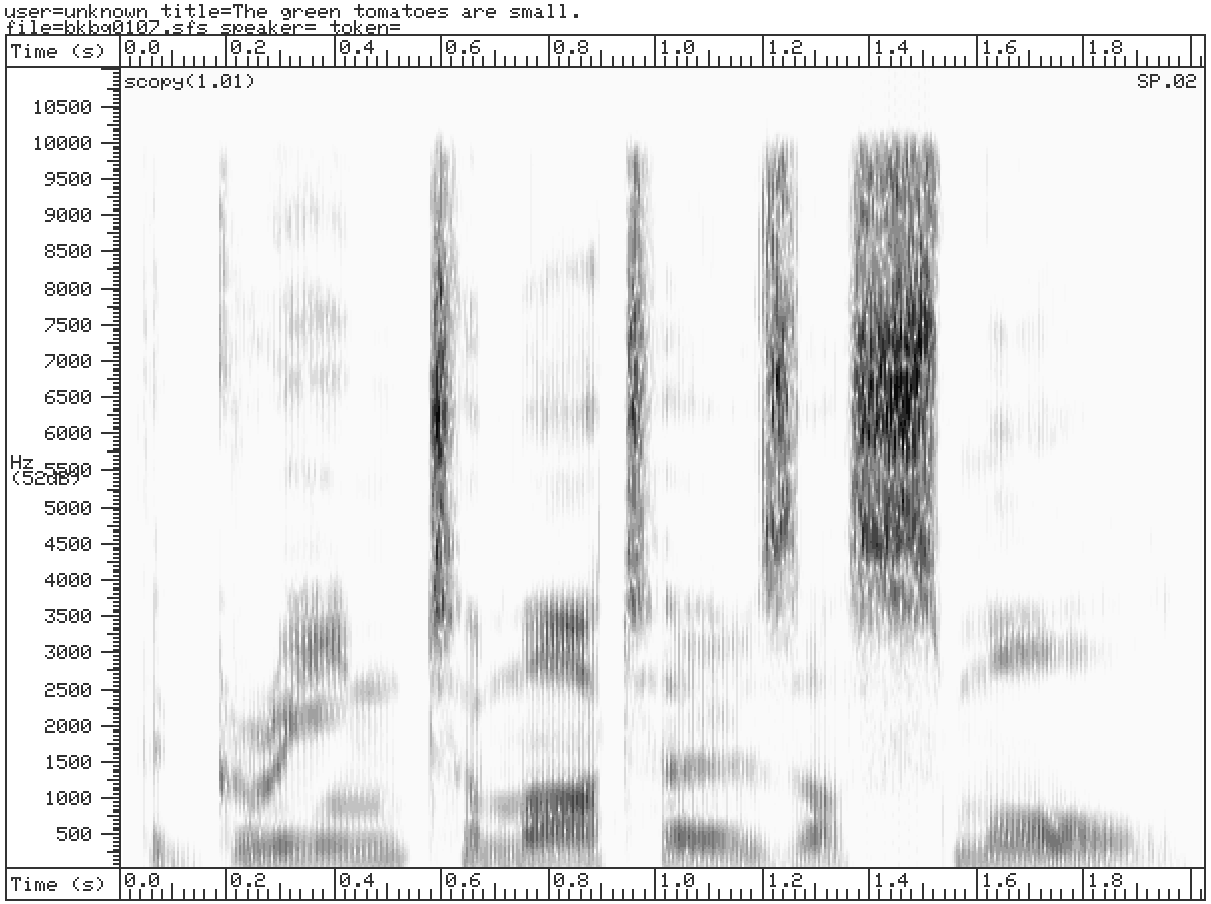


What can you use filter banks for?

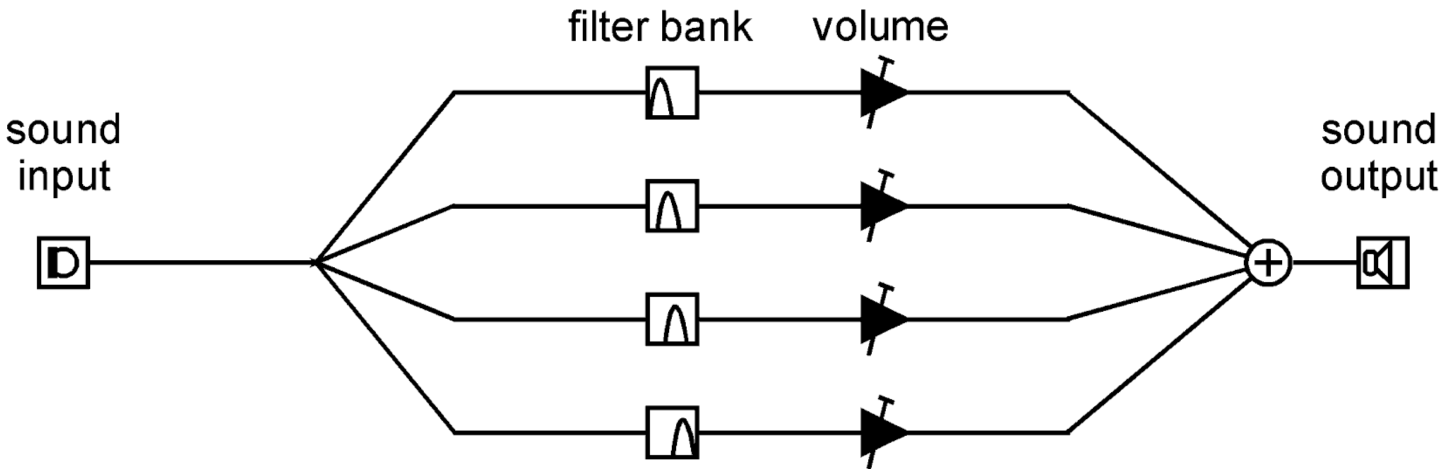


Other than spectral analyses ...

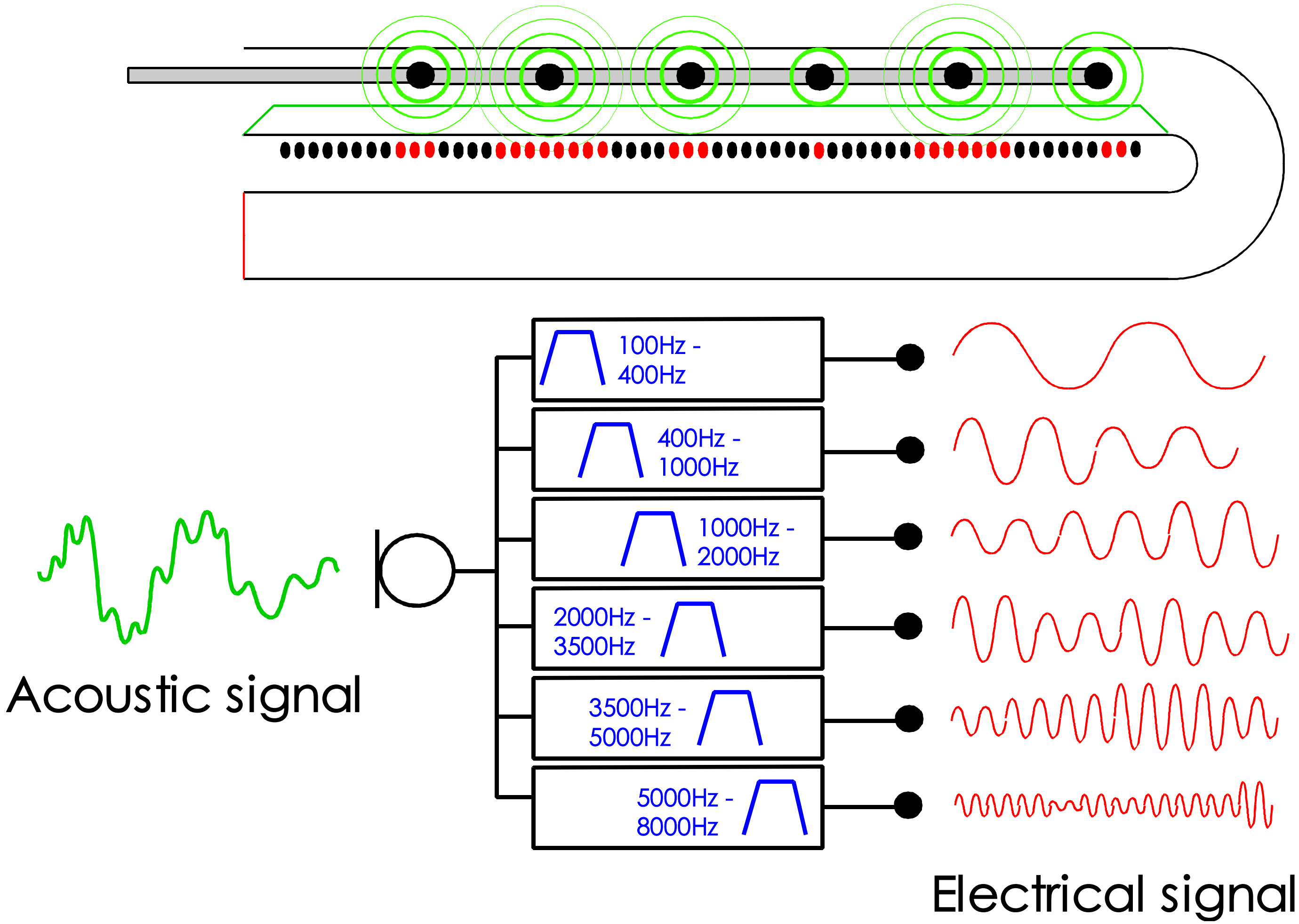
To make spectrograms or voiceprints ...



To make a graphic equaliser …



To process sounds for a multi-channel cochlear implant (an electronic filter bank substitutes for the basilar membrane)



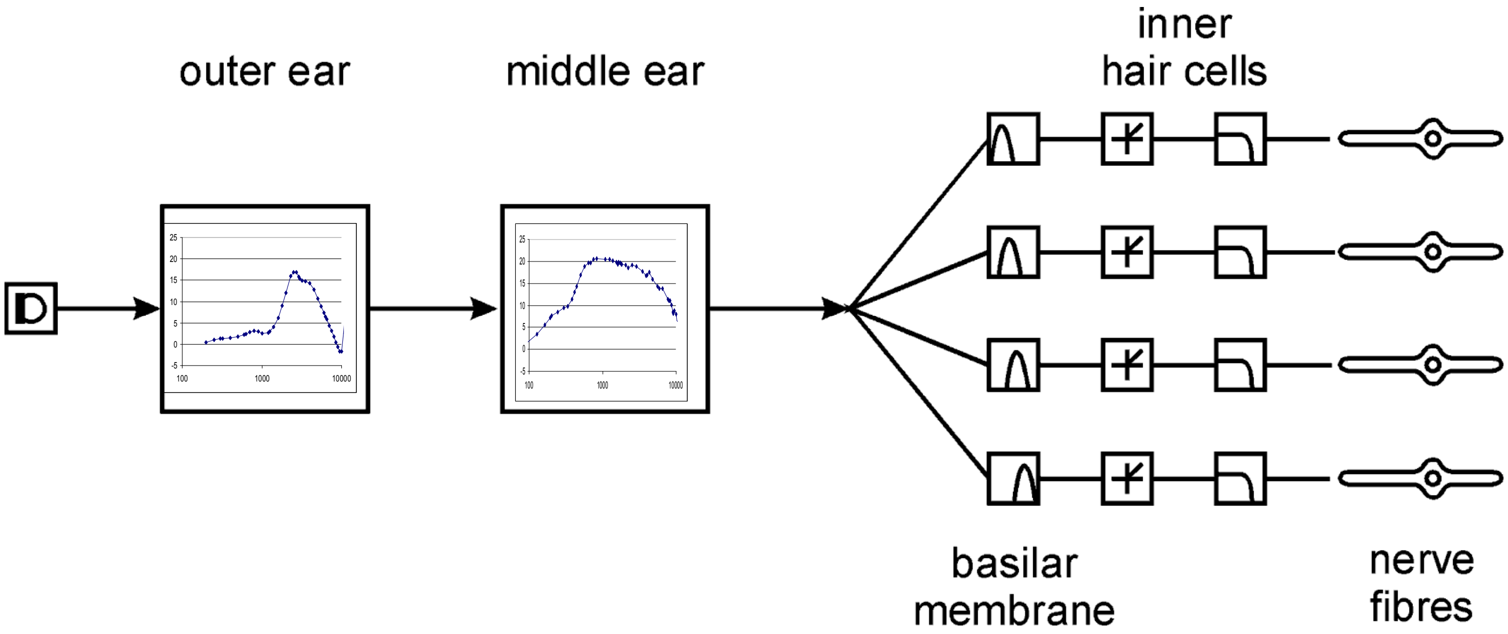
In hearing aids ...

Shape the spectrum of incoming sounds to compensate for the hearing loss

frequency regions with bigger loss get greater gain

a graphic equaliser!

In computational models of the auditory periphery.



Imagine that each afferent auditory nerve fibre has a bandpass filter attached to its input.